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Analysis of bounds for a capacitated single-item lot-sizing problem

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Abstract

Lot-sizing problems are cornerstone optimization problems for production planning with time varying demand. We analyze the quality of bounds, both lower and upper, provided by a range of fast algorithms. Special attention is given to LP-based rounding algorithms.

1. Introduction

Lot-sizing problems have attracted attention for decades because they form the cornerstone optimization problems for production planning with time varying demand (the economic order quantity model being the cornerstone optimization problem for production planning with stationary demand). One of the chief difficulties in solving lot-sizing problems arises from capacity restrictions in each time period. In fact, the problem is polynomially solvable when capacities are absent [1], as well as when production capacities are identical in each period of the planning horizon [2]. When capacities are allowed to vary over the planning horizon, however, the problem is NP-hard.

Lot-sizing problems are well studied. A good exact approach for the single-item lot-sizing problem with general capacities comes from Chen & Lee [3], who provide a dynamic program to solve it. Polyhedral

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results for these problems are especially plentiful. Notable results are found in, among others, [4–6] (see [7] for a more comprehensive review). The abundance of polyhedral results has resulted in more effective integer programming based solution approaches, but many instances still require prohibitive amounts of computation time. Van Hoesel and Wagelmans [8] approach the problem differently and present a fully polynomial approximation scheme for single-item capacitated lot-sizing, with the time required to find a solution depending on the desired nearness of the value of the resulting solution to the optimal value. Unfortunately, the practical value of this approach is limited as the computational requirements are high even for moderate approximation factors.

Therefore, we focus our efforts on developing quick procedures that provide quality bounds (both lower and upper) for lot-sizing problems. We pared the problem down to its simplest form by assuming the absence of unit production and holding costs. The only factors then influencing solution feasibility and quality are the setup costs and production capacities. By doing so, we hope to gain better understanding of the interaction between setup costs and capacities and how they influence the quality of bounding and solution techniques. This variant of the lot-sizing problem is polynomially solvable when the instance data satisfy certain conditions. For instance, when capacities are nonincreasing and setup costs are nondecreasing over time, then the problem can be easily solved. However, when capacities and setup costs are allowed to vary arbitrarily the problem is NP-hard (see [9] for a more thorough discussion of complexity results for capacitated lot-sizing).

Another motivation for studying approximation algorithms for lot-sizing problems is the recent success of LP-based approximation algorithms for a variety of discrete optimization problems [10], in particular scheduling problems ([11–15], to name a few). Given the serial component common to both scheduling and lot-sizing problems, it is natural to investigate the potential of LP-based techniques for lot-sizing problems.

We begin, in Section 2, by formally defining the problem we consider, by identifying assumptions we make throughout the paper, and by presenting the integer programming formulation on which our studies are based. Then, in Section 3, we examine a special case of the problem, providing a polynomial-time algorithm to solve it. Subsequently, in Section 4, we return to the general case and study various lower bounds. We start by presenting an algorithm to solve the LP relaxation in quadratic time. Next, we analyze the strength of the LP relaxation, of a ratio-based lower bound inspired by our algorithm to solve the LP relaxation, and of a constant capacity relaxation. Following the study of lower bounds, we introduce, in Section 5, several upper bounds based on LP rounding and one upper bound inspired by our algorithm to solve the LP relaxation, offering a worst-case analysis of each.

The performance guarantees derived are data dependent. The typical guarantee is that the relative deviation from the optimal value is not larger than the maximum capacity. Even though such guarantees may be weak, they are shown to be tight. Therefore, it appears unlikely that the success of LP-based approximations algorithms can easily be replicated in the context of lot-sizing problems.

2. Problem description (LSZIP)

For each period t in the discrete planning horizon $\{1, \ldots, T\}$, we are given demand d_t, production capacity C_t, and cost f_t which is incurred if any positive level of production takes place in period t. The objective, then, is to schedule production that respects all capacities and satisfies all demands, where production in period t may be used to satisfy demand in any of the periods $\{t, \ldots, T\}$. We denote this