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Permeability prediction of an analytical pore-scale model for layered and isotropic fibrous porous media

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HIGHLIGHTS

- Geometric models for 2D layered and isotropic 3D fibrous porous media are proposed.
- The permeability for through plane and in-plane flow in layered fibres are predicted.
- Effects of pore blockage and developing flow are successfully accounted for.
- Prediction accuracy depends on model assumptions and velocity ranges involved.
- The model equations contain no empirical coefficients and are physically adaptable.

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ABSTRACT

In this study an existing analytical pore-scale model for flow parallel and perpendicular to 1D unidirectional fibres are used and adapted to propose permeability predictions for in-plane and through plane flow in layered 2D fibre arrangements as well as flow through 3D isotropic fibrous porous media. This is done by application of a weighted average equation in which different weights are assigned to flow parallel and perpendicular to 1D fibre arrangements, depending on the fibre orientation and average flow direction. Different arrays are considered, based on the degree of staggering of fibres with respect to the average flow direction. The effect of fibre orientation on permeability is investigated for flow through layered 2D fibre arrangements. It is illustrated how the permeability at low solid volume fractions reduces through the incorporation of the effect of developing flow. The effect of blocked pores at high solid volume fractions is also accounted for by introducing a percolation threshold solid volume fraction beyond which no more seepage takes place. A unimodal equivalent radius is furthermore introduced into the models in order to predict the permeability of bimodal fibrous media. The proposed permeability predictions are applicable over the entire range of solid volume fractions. Comparison with theoretical models from the literature as well as available experimental and numerical data proves satisfactory. The model characteristics that distinguish the proposed models from many other models in the literature are (i) being physically adaptable to extend its range of applicability, whilst at the same time (ii) balancing accuracy and simplicity (from an analytical point of view), and (iii) containing no empirical coefficients.

1. Introduction

Fibrous porous media find application in many industries including the paper production and textile industry, filtration processes, fuel cells, acoustics, compact heat exchangers and biological systems (Soltani et al., 2014; Stylianopoulos et al., 2008). The advantages of these materials are their mechanically strong and stable microstructures at low solid volume fractions. These characteristics, including the high permeability, are what makes these materials so attractive for the wide variety of industrial applications (Jaganathan et al., 2008b; Tahir and Tafreshi, 2009; Zobel et al., 2007). As a result, the transport properties of these media are important. Several modelling strategies have therefore been proposed towards predicting the permeability of fibrous porous media in view of obtaining a better understanding of the underlying physical flow phenomena.

The first theoretical studies were based on solving the Stokes equations together with the introduction of a conceptual model subject to appropriate boundary conditions. The conceptual models mostly comprise of a cylindrical unit cell involving flow through ordered arrays (including square, triangular and hexagonal) of unidirectional cylinders (e.g. Happel, 1959; Kuwabara, 1959; Sangani

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and Acrivos, 1982). Drummond and Tahir (1984) added terms to the permeability prediction of Sangani and Acrivos (1982) by making use of a matching technique in the cylindrical unit cell approach. Spielman and Goren (1968) regarded a unit cell to be surrounded by an infinite homogeneous fibrous porous medium and therefore solved an equation consisting of a superposition of the Stokes equation and Darcy's law in order to predict the permeability. Another conceptual modelling approach involving Fourier series applied to square arrays of simple, face-centered and body-centered lattices was proposed by Hasimoto (1958). A detailed summary of these models as well as a comprehensive collection of experimental data from numerous authors (e.g. Bergelin et al., 1950; Kirsch and Fuchs, 1967), based on various types of fibrous porous media, are provided by Jackson and James (1986) and more recently by Tomadakis and Robertson (2005) and Tamayol and Bahrami (2010).

With the advancement of technology in recent years, numerical studies on flow through fibrous porous media increased significantly. These studies include the pioneering work of Tahir and Tafreshi (2009) and Jaganathan et al. (2008b). Many authors have artificially constructed fibre networks obtained by techniques such as magnetic resonance imaging, X-ray micro-computed tomography (e.g. Soltani et al., 2014) and digital volumetric imaging (e.g. Jaganathan et al., 2008b). The flow is simulated through such networks by solving the Stokes equations in combination with numerical methods such as the finite element method (Stylianopoulos et al., 2008), the Lattice Boltzman method (e.g. Nabovati et al., 2009), the spectral boundary element method (e.g. Higdon and Ford, 1996) and boundary element method (Chen and Papathanasiou, 2006; Papathanasiou, 2001).

Although experimental studies are, in general, time-consuming and expensive to perform (Soltani et al., 2014), the data provided are extremely useful for the validation of numerical and analytical models. Van Doormal and Pharoah (2009), on the other hand, stresses the important role that permeability predictions, obtained from analytical models, play in computational fluid dynamics models. In recent years several authors have combined numerical and analytical or empirical modelling procedures. Stylianopoulos et al. (2008), for instance, have combined their numerical approach with the volume averaging method in order to determine the permeability for their drag model. Nabovati et al. (2009) made use of their numerical data to adjust the coefficient values of the combined theoretical, numerical and empirical model of Gebart (1992). Tahir and Tafreshi (2009) used the empirical permeability of Davies (1952) in their numerical simulations in order to investigate the effect of fibre orientation on permeability and Gostick et al. (2006) used the model of Tomadakis and Robertson (2005) in their simulations. The latter authors followed an analytical electrical conduction-based method in order to predict the permeability of arrays of cylindrical fibres.

Different categories of flow through uniform and straight fibrous porous media have been considered in the literature depending on fibre orientation, flow direction and application. The categories to be considered in this study are the following: (i) flow through arrays of unidirectional fibrous porous media, referred to as a one-dimensional (1D) fibre arrangement and subdivided into (a) flow parallel to the axes of unidirectional fibres and (b) flow perpendicular to the axes of unidirectional fibres, also referred to as transverse flow; (ii) flow through fibres of which the axes are randomly distributed in a plane, referred to as a two-dimensional (2D) layered fibre arrangement and subdivided into (a) in-plane flow, i.e. flow parallel to the plane in which the fibres lie and (b) through plane flow, i.e. flow perpendicular to the plane in which the fibres lie and lastly (iii) flow through randomly distributed fibres in space, referred to as a three-dimensional (3D) fibre arrangement. Categories (i) and (ii) are anisotropic, although the latter category can be regarded as transversely isotropic with respect to the plane in which the fibres are randomly distributed (Feser et al., 2006). Category (iii) is regarded as isotropic. The earlier studies were mostly concerned with flow through 1D fibre arrangements (e.g. Drummond and Tahir, 1984).

Jackson and James (1986), in their collective study, considered categories (i) (a)&(b) and (iii), whereas Spielman and Goren (1968) considered all the categories. Tamayol and Bahrami (2011b), Stylianopoulos et al. (2008) and Tomadakis and Robertson (2005) have also studied 1D, 2D and 3D fibre arrangements. The latter authors in addition allowed for overlapping of fibres.

2D layered fibrous media became of great importance in the filtration industry in which the filter paper is made up of fibres that have been deposited onto a flat surface so that the fibres' axes are randomly distributed onto planes parallel to the surface (Spielman and Goren, 1968). As a result of this, more attention has been drawn by authors towards studying the permeability of these media (e.g. Gostick et al., 2006; Soltani et al., 2014; Tahir and Tafreshi, 2009; Tamayol and Bahrami, 2011a; Tamayol and Bahrami, 2011b, Tamayol et al., 2012).

Tamayol and Bahrami (2011b) have provided equations for predicting the permeability for all the categories (based on a scale analysis technique), but introduced empirical coefficients in each of them to obtain correlation with experimental data. The objective of this study is to propose an adaptable analytical model based on physical principles (i.e. excluding any empirical coefficients) in order to predict the permeability of fibrous media in each of the categories mentioned above as a function of solid volume fraction and uniform fibre radius. The fibre geometry will be assumed rectangular in shape, similarly as was done by Zobel et al. (2007). The novelty is that 2D models for through plane and in-plane flow will be based on the existing fibre Representative Unit Cell (RUC) models for flow through 1D fibre arrangements. A 3D fibre model will also be proposed and in addition to the 2D models adapted to account for secondary effects such as developing flow that becomes significant at low solid volume fractions and pore blockage at high solid volume fractions. The effect of fibre orientation will be investigated for the RUC models resembling 2D layered fibrous media and the models furthermore adapted to incorporate a unimodal equivalent radius in the case of bimodal fibrous media.

2. Flow through 1D fibre arrangements

The rectangular RUC model for fibrous porous media was initially introduced by Du Plessis (1991). The single rectangular fibre in a rectangular unit cell, shown in Fig. 1, is representative of the average pore-scale geometry of a 1D fibre arrangement. The solid width is denoted by d_s, with L being the fibre length and d the cell size.

The RUC modelling approach differs from the unit cell approach (of e.g. Tamayol and Bahrami, 2011b) in which the unit cell is repeated throughout the porous medium in order to periodically construct the medium. The RUC model is not a building block, but rather the smallest rectangular unit cell in which the *average* geometrical properties of a Representative Elementary Volume (REV) can be embedded to be *representative* of the actual pore-scale geometry (Du Plessis and Masliyah, 1988). The total solid and fluid volumes are denoted by U_s and U_f, respectively and $U_o = d^2 L$ denotes the total volume of the RUC. The porosity is therefore defined as

$$\epsilon = \frac{U_f}{U_o}, \tag{1}$$