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# Characterization of stability limits of Ledinegg instability and density wave oscillations for two-phase flow in natural circulation loops

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## HIGHLIGHTS

- Interaction of Ledinegg instability and density wave oscillations (DWOs) is studied.
- Bogdanov-Takens bifurcation point is identified as transition from DWO to Ledinegg.
- Cusp point is identified as lower limit of subcooling number for Ledinegg to occur.
- Saddle node and Hopf bifurcation are identified as Ledinegg and DWOs, respectively.
- Regions in parameter space associated with different instabilities are presented.

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## ABSTRACT

A detailed study of excursive (or Ledinegg) instability and density wave oscillations (DWOs) is carried out for two-phase flow in a natural circulation loop. The maps in the parametric space have been obtained, which indicate excursive instability (Ledinegg instability) as well as density wave oscillations (DWOs). The dynamic (DWOs) and static (Ledinegg) stability boundary on these maps have been drawn. These diagrams dealing with the interaction of Ledinegg instability and DWOs have rarely been reported, and detailed mathematical analysis of these maps is lacking. In the present work, a detailed study of such interactions is carried out. It is also noted that the Ledinegg instability is a manifestation of saddle node bifurcation or limit point (LP) for the dynamical system of natural circulation loop. Density wave oscillations in the dynamical system of natural circulation loop can be observed with the Hopf bifurcation. The detailed bifurcation analysis of these instabilities shows that stability maps can be divided into three broad regions, the first region can only have DWOs (Hopf bifurcation), and the second region can have Ledinegg as well as DWOs (Hopf bifurcation and saddle node bifurcation co-exists), while the third region can have only Ledinegg instability (saddle node bifurcation). The first and second region are separated by a Cusp point (CP) and it is found that between CP and Bogdanov-Takens (BT) bifurcation point both Hopf and LP exist, due to presence of both DWOs and Ledinegg instability. The region beyond BT bifurcation point only has Ledinegg or excursive instability. Moreover, DWOs in the first region have two types of Hopf bifurcation depending on the nature of limit cycles. These subcritical and supercritical Hopf bifurcations, are separated by the Generalized Hopf (GH) bifurcation. Abovementioned three regions are observed only for the Type II DWOs stability boundary, whereas Type I DWOs have only subcritical and supercritical Hopf bifurcations. The impact of design parameters on the saddle-node curve along with the shifting of interaction point (Bogdanov-Takens bifurcation) have been investigated as well.


## 1. Introduction

The instabilities relevant for two-phase flow systems received the extensive attention of researchers in recent decades due to its growing applications in the industries such as nuclear, oil, refrigeration, turbo machinery, boiler, steam generators, thermosyphons, economizer, etc. The two-phase flow instabilities in natural circulation loops have also been studied in recent decades due to the advantage of passive heat removal and better heat transfer characteristics of the natural circulation loops. Natural circulation loops are an important passive heat removal mechanism for an Advanced Boiling Water Reactor during the startup and shutdown processes of the reactor. In particular, the state under natural circulation is closer to the unstable region, the study of natural

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**Nomenclature**

| | |
|---|---|
| a | identifying coefficient for LP and BT |
| $A^*$ | cross section area ($m^2$) |
| $a_1(t)$ | $\frac{N_\rho N_r N_{pch}}{v_{inlet} * \rho_{inlet}}$ |
| b | identifying coefficient for BT |
| $B_t^*$ | loop width (horizontal section, m) |
| $C_o$ | void distribution parameter |
| $D_h^*$ | hydraulic diameter of flow channel (m) |
| $D_{doc}^*$ | diameter of downcomer (m) |
| $D_r^*$ | diameter of riser (m) |
| $f$ | friction factor, $\frac{0.184}{R_e^{0.2}}$ |
| $Fr$ | Froude number, $\frac{v_0^{*2}}{g^* L_{ch}^*}$ |
| $g^*$ | acceleration due to gravity ($m/s^2$) |
| $h^*$ | enthalpy (kJ/kg) |
| $H_t^*$ | downcomer level (m) |
| $j^*$ | volumetric flux (m) |
| $J$ | Jacobian matrix |
| $k_{inlet}$ | inlet pressure loss coefficient |
| $k_{exit}$ | exit pressure loss coefficient |
| $L_{ch}^*$ | total length of flow channel (m) |
| $L_r^*$ | length of riser (m) |
| $N_f$ | friction number $\frac{fL_{ch}^*}{2D_h^*}$ |
| $N_{pch}$ | phase change number $\frac{q''^* \Delta\rho^* \xi_h^* L_{ch}^*}{A^* \Delta h_{fg}^* v_0^* \rho_g^* \rho_f^*}$ |
| $N_{sub}$ | subcooling number $\frac{(h_{sat}^* - h_{inlet}^*)\Delta\rho^*}{\Delta h_{fg}^* \rho_g^*}$ |
| $N_{feed}$ | feed number $\frac{(h_{sat}^* - h_{feed}^*)\Delta\rho^*}{\Delta h_{fg}^* \rho_g^*}$ |
| $q''^*$ | wall heat flux ($W/m^2$) |
| $s_1(t)$ | $\frac{N_\rho N_r N_{pch}}{v_{inlet} * \rho_{inlet}}$ |
| $t^*$ | time (s) |
| $v_0^*$ | reference velocity (m/s) |
| $v_{inlet}^*$ | inlet velocity of coolant (m/s) |
| $V_{gj}^*$ | average drift velocity (m/s) |
| $x$ | steam quality |
| $z^*$ | distance along the axis of flow channel (m) |
| $\xi_h^*$ | heated perimeter (m) |
| $\alpha$ | void fraction |
| $\mu^*$ | boiling boundary (m) |
| $\rho^*$ | density ($kg/m^3$) |

*Subscripts*

| | |
|---|---|
| $1\Phi$ | single phase |
| $2\Phi$ | two phase |
| *doc* | downcomer |
| *ex* | exit |
| *f* | liquid |
| *g* | vapor |
| *in* | inlet |
| *m* | mixture |
| *r* | riser |

*Superscripts*

| | |
|---|---|
| * | dimensional quantity |

*Abbreviations*

| | |
|---|---|
| AHWR | advanced heavy water reactor |
| BT | Bogdanov-Takens bifurcation |
| BWR | boiling water reactor |
| CP | cusp point bifurcation |
| DWO | density wave oscillation |
| FLC | first Lyapunov coefficient |
| GH | generalized hopf bifurcation |
| H | Hopf Bifurcation |
| LP | limit point Bifurcation |
| LPC | limit point bifurcation of limit cycles |

circulation is, therefore, very important for an Advanced Boiling Water Reactor.

The study of these, two phase instabilities have been categorized in work of (Bouré, 1973) as static and dynamic instabilities. The stability behavior of the static instabilities can be defined from the steady states of the system. The dynamic instabilities occur due to the dynamic effects of the system such as propagation time, compressibility, etc. The detailed reviews of this two phase flow instabilities in different systems have been documented (Durga Prasad and Pandey, 2008; Kakac and Bon, 2008; Nayak et al., 2007; Ruspini et al., 2014; Tadrist, 2007).

Density wave oscillations, pressure drop oscillations and excursive instabilities (i.e. a type of Ledinegg instability) are the most common instabilities occurring in the two-phase natural circulation loops. Density wave oscillations are a dynamic instability, whereas excursive instability is a static instability. The mechanisms associated with density wave oscillations are the delay in propagation of perturbations and feedback effects to the initial parameters of the concerned system. The change in the inlet flow rates affects the void generation, and hence the two-phase mixture density. The propagation of the change from inlet to the outlet with delay, changes the pressures drop across the channel and intensifies disturbance in the flow rate. This feedback of flow rate in the loop is manifested as density wave oscillations in the loop (Bouré, 1973; Rizwan-Uddin, 1994). The classification of density wave oscillations based on the dynamics and mechanisms associated with the oscillations is given in (Fukuda and Kobori, 1979). The Type I and Type II density wave oscillations are very common in natural circulation loops. The major contribution of gravity head in the pressure drop defines the Type I DWOs whereas the major contribution of two-phase flow pressure drops in the overall pressure drop leads to Type II DWOs.

Linear and nonlinear stability analysis of DWOs have been carried out with three approaches, frequency domain analysis, time domain analysis, and bifurcation analysis. In frequency domain analysis, the control techniques such as Laplace transformation is used for the linearized model of the governing equations. The implementation of this technique to study flow instability for the two-phase flow natural circulation loop is discussed in the work of (Lee and Lee, 1991). The occurrence of excursive instability is mentioned in this work as well, although, its interaction with the bifurcations is not covered in this work. The limitation of frequency domain analysis is that; one cannot investigate nonlinear stability analysis. Hence bifurcation analysis cannot be done with this approach. However, time domain analysis and bifurcation analysis of the reduced order model allows nonlinear stability analysis for the natural circulation loops. A linear and nonlinear analysis of density wave oscillations with is given in work of (Durga Prasad and Pandey, 2008), and detail review of DWOs in natural circulation loops can be found in (Nayak and Vijayan, 2008).

The density wave oscillations are mathematically represented by Hopf bifurcation, and nonlinear stability analysis predicts limit cycles near to this bifurcation. Self-sustained oscillations of DWOs are described mathematically in terms of limit cycles, which can be stable or unstable depending on the type of Hopf bifurcation. Limit