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A paradox in optimal flow control of M/M/n queues

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Abstract

Optimal flow control problems of multiple-server (M/M/n) queueing systems are studied. Due to enhanced flexibility of the decision making, intuitively, we expect that grouping together separated systems into one system provides improved performance over the previously separated systems. This paper presents a result counter-intuitive against such an expectation. We consider a non-cooperative optimal flow control scheme M/M/n queueing systems where each of multiple players strives to optimize unilaterally its own power where the power of a player is the quotient of the throughput divided by the mean response time for the player. We report a counter-intuitive case where the power of every user degrades after grouping together $K(>1)$ separated M/M/N systems into a single M/M/$(K \times N)$ system. Some numerical results are presented.

Keywords: Braess paradox; Flow control; Nash equilibrium; M/M/n queueing system; Non-cooperative game; Optimization

1. Introduction

In computer and communication networks, flow control is one of the important means to utilize limited system resources effectively and to guarantee proper quality of service (QoS). Flow control adjusts the input flow (throughput) in order to provide good performance. Considering an optimal flow control problem, one may face the trade-off between the throughput and the response time. These two performance measures are mutually contradictory, that is, if one improves the system throughput, then the system response time degrades, and vice versa. Therefore, as the utility of each user (player), we use the power

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that is the quotient of the throughput divided by the average response time for the user (see, e.g., Giessler et al. [1], Kleinrock [23]).

We consider a system where multiple users (players) share an M/M/n queueing system and where the utility of a player is the power. We can think of two typical performance optimization schemes: the *non-cooperative optimization* scheme and the *overall optimization* scheme. In the non-cooperative optimization scheme, each player strives to optimize unilaterally its own power given the decisions by others. In the overall optimization scheme, a single performance measure that is the total sum of the powers of all players is optimized by a single agent. The former is regarded as a non-cooperative game, and the equilibrium is called a *Nash equilibrium*. In this paper, we study the *Nash equilibrium* [4].

Douligeris and Mazumdar [5], and Zhang and Douligeris [6] studied algorithms to obtain Nash equilibria in flow control of M/M/1 queueing systems with multiple users. Their performance objective was to maximize the power. They proposed greedy algorithms, and showed convergence properties of them. State-dependent flow control was analyzed by Hsiao and Lazar [7], and Korilis and Lazar [8]. They considered a closed queueing network model, and maximized the average throughput subject to an upper bound on the average response time. In particular, Korilis and Lazar [8] derived the existence of equilibria using fixed-point theorems. Altman et al. [9] combined flow control and routing in a network model with several parallel links. Lazar [10] studied optimal flow control problems of an M/M/n queue where one player maximizes the throughput subject to the constraint that the average time delay should not exceed a specified value. In this paper, we deal with flow control problems of M/M/n queueing systems that have multiple players, where each player strives to optimize unilaterally its power.

Consider the operation of grouping together separated systems into one system. Due to enhanced flexibility in resource utilization, we expect that system grouping improves the system performance. For example, it has been shown analytically that grouping together separated systems improves the average response time although the utilization factor of each server remains the same (see [11]). Therefore, we expect performance improvement in terms of power by grouping together separate systems.

In noncooperative optimization of routing in networks and load balancing in distributed systems, however, it is known that the following phenomenon can occur: Grouping separated systems together and/or adding connections to the system may sometimes degrade the utilities for all users. The flrst example is the Braess paradox [12]. Other examples have been presented, for example, by Bean et al. [13], Calvert et al. [14], Cohen and Jeflries [15], Cohen and Kelly [16], Kameda et al. [17], Kameda and Pourtallier [18], Korilis et al. [1920], Roughgarden and Tardos [21]. However, most of the previous results are on routing (or load balancing) problems, and the paradox is seen in very limited models. Flow control is essentially different from them. In optimal flow control, it seems that no such paradoxes have been reported yet.

In this paper, we show that a paradox similar to the above ones may occur in noncooperative optimal flow control of M/M/n queues. That is, we show a case where grouping leads to the degradation of the power to all players in noncooperative optimal flow control.

The rest of this paper is organized as follows. In Section 2, we describe a queueing system model and formulate overall and noncooperative optimal flow control schemes as nonlinear programming problems. In Section 3, we show the case of a paradox in noncooperative flow control of M/M/n queueing systems. Finally, in Section 4, we conclude this paper. In the Appendix, we show the existence and the uniqueness of solutions to the optimization problems, and present algorithms that obtain the solutions.