Available online at www.sciencedirect.com
SCIENCE DIRECT®
Computers & Operations Research 32 (2005) 3111–3128
computers & operations research
www.elsevier.com/locate/cor

New heuristic algorithms for the windy rural postman problem

Enrique Benavent[a,*], Angel Corberán[a], Estefanía Piñana[a], Isaac Plana[a], José M. Sanchis[b]

[a]Departmento d'Estadística i Investigació Operativa, Universitat de València, Dr. Moliner 50, Burjassot, Valencia 46100, Spain
[b]Departmento de Matemática Aplicada, Universidad Politécnica de Valencia, Spain

Available online 24 June 2004

Abstract

In this paper we deal with the windy rural postman problem. This problem generalizes several important arc routing problems and has interesting real-life applications. Here, we present several heuristics whose study has lead to the design of a scatter search algorithm for the windy rural postman problem. Extensive computational experiments over different sets of instances, with sizes up to 988 nodes and 3952 edges, are also presented.

Keywords: Heuristics; Metaheuristics; Arc routing; Rural postman problem; Windy rural postman problem

1. Introduction

In the well known (undirected) Chinese postman problem (CPP) the cost of traversing an edge (i, j) from i to j is the same as that of traversing it from j to i. If, as in some real-world applications, this is not the case, the problem is called the windy postman problem (WPP). The WPP was proposed by Minieka [1], who named this problem illustrating the work that a postman should perform on a windy day.

It is easy to see that, by a proper definition of the edge costs, the undirected, directed and mixed Chinese postman problems are special cases of the WPP. Therefore, since the mixed Chinese postman problem (MCPP) is *NP*-hard, the WPP is also a *NP*-hard problem [2,3], although some particular cases can be solved in polynomial time [3,4]. An integer linear programming formulation of the WPP was given in

* Corresponding author. Tel.: +34-96-386-4354; fax: +34-96-386-4735.
E-mail address: enrique.benavent@uv.es (E. Benavent).

doi:10.1016/j.cor.2004.04.007

Grötschel and Win [5]. In that paper, the authors also proposed a cutting plane algorithm, based on a partial description of the windy postman polyhedron, producing good computational results. Concerning the approximate resolution of the WPP, several heuristic algorithms have been proposed by Win [6] and Pearn and Li [7].

In this paper we deal with a generalization of the WPP, the windy rural postman problem (WRPP). This problem can be described as follows. Let $G = (V, E)$ be an undirected and connected graph with two nonnegative costs, c_{ij}, c_{ji}, associated to each edge $(i, j) \in E$, representing the cost of traversing the edge from i to j and from j to i, respectively. Let E_R be a subset of edges of E, that will be called the required edges set. Then, the WRPP is the problem of finding a minimum cost closed walk traversing every edge in E_R at least once.

Obviously, if $E_R = E$, the WRPP reduces to the WPP, but note that the WRPP also generalizes the undirected, directed and mixed rural postman problems. Therefore, the WRPP is a difficult *NP*-hard problem that contains as special cases most of the known arc routing problems with a single vehicle. Moreover, besides its great theoretical interest, the WRPP has some very interesting practical applications (see [8]).

Up to the authors knowledge, the only paper dealing with the WRPP is that of Benavent et al. [8]. In their paper, authors propose an integer linear programming formulation and different heuristics and lower bounds for the WRPP. Lower bounds are obtained by means of a cutting plane procedure that, besides using the inequalities in the WRPP formulation, it also identifies other violated valid inequalities coming from families of facet defining inequalities for the RPP polyhedron. The upper bounds are computed by means of three constructive heuristics and several improvement procedures. The computational results are very good. The cutting plane algorithm was able to solve 185 out of 288 tested instances. For the unsolved instances, the average deviation from the optimal value is always less than 1%. On the other hand, the best solution among those obtained with the 3 heuristic procedures proposed in Benavent et al. [8] is 4% on average from the lower bound and it is computed in less than 1 second.

In what follows we will assume, for the sake of simplicity and without loss of generality, that all the nodes in V are incident with required edges. From the original graph $G = (V, E)$, let G_R be the graph induced by the required edges, i.e. $G_R = (V, E_R)$. Graph G_R is in general not connected. The sets of nodes of the connected components of G_R are usually called R-sets. A WRPP solution will be a strongly connected directed multigraph $G^* = (V, A)$ satisfying that: every node in G^* is symmetric, each arc $(i, j) \in A$ is a copy of an edge in E with a given orientation, and for each $(i, j) \in E_R$, either (i, j), (j, i), or both belong to A.

It is well known that there exists a closed walk (tour) traversing each arc of G^* exactly once. This tour is an alternative way of representing a WRPP solution.

In this paper, we present new heuristic algorithms which have a very good computational behavior on different sets of instances. In Section 2 we summarize two of the algorithms described in Benavent et al. [8] and present two new constructive algorithms which are modified versions of them. Section 3 describes several improvement procedures, while Section 4 describes four multi-start procedures that have been obtained by randomizing certain steps of the constructive algorithms. Finally, a scatter search algorithm that combines the best previous procedures is presented in Section 5. Computational results for each class of heuristics are included in their respective section. We have tried a large set of instances including those used by Benavent et al. [8] and also some new large instances that have been randomly generated. Some conclusions are presented in Section 6.