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Solving the hub location problem with modular link capacities

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Abstract

This paper deals with a capacitated hub location problem arising in the design of telecommunications networks. The problem is different from the classical hub location problem in two ways: the cost of using an edge is not linear but stepwise and the capacity of a hub restricts the amount of traffic transiting through the hub rather than the incoming traffic. In this paper both an exact and a heuristic method are presented. They are compared and combined in a heuristic concentration approach to investigate whether it is possible to improve the results within limited computational times.

Keywords: Hub location; Branch and cut; Tabu search; Heuristic concentration

1. Introduction and problem description

Optimization problems related to the design and installation of telecommunication networks gain interest due to the growing importance of telecommunications. Since nowadays installing telecommunication networks is expensive, an optimization phase with the goal of minimizing costs is needed in the design process. This paper deals with an optimization problem arising in the design of a quite common network architecture, the so-called *backbone/tributary network*, in which two kinds of nodes are present. *Terminal nodes* represent origins and destinations of the traffic demands to be routed. Usually, connecting all pairs of terminals nodes by direct links is a very costly solution. So, the traffic originating in different terminal nodes must be collected in nodes called *hubs*, which receive

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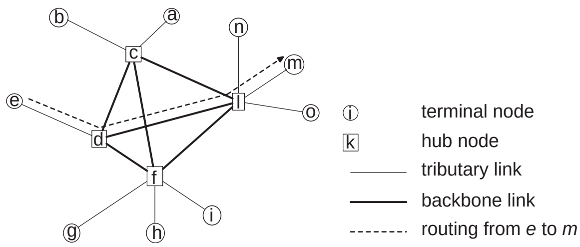

Fig. 1. Backbone/tributary network instance.

traffic from terminal nodes and route it through other hubs towards the destinations. The network connecting the terminal nodes to the hub to which they are assigned is called the *tributary network* and the network connecting the hubs is called the *backbone network*. Since the cost per unit of traffic is usually cheaper in the backbone than in the tributary network, collecting traffic allows to reduce the costs.

Different kinds of optimization problems may arise in backbone/tributary network design. Klincewicz [1] gives a survey on these problems and Yuan [2] gives an annotated bibliography in communication network design and routing problems that presents over 600 references. Problems differ in many aspects, e.g. it is possible to focus on the design of the backbone and/or the tributary networks or on the hub location problem. Furthermore, it is possible to consider different kinds of costs, e.g. to consider fixed costs for installing hubs and/or variable costs for using them as well as fixed costs for installing the needed capacity on the edges or variable costs for using the edges. It is possible to limit the capacities of both edges and hubs. Moreover, in some problems, an a priori structure is specified for the networks. For example, one can look for a network where the backbone is a tree and the tributary networks are stars.

In this paper, we focus on networks with complete backbone and star tributary networks and solve the location and dimensioning problems simultaneously. The locations of the hubs are chosen among the terminal nodes and each terminal node is assigned to exactly one hub. All the outgoing traffic of a terminal node is sent to the hub to which it is assigned, and is routed through the other hubs towards the destinations. All the traffic incoming in the terminal node is routed from its origins through the hubs towards the hub to which the node is assigned. The hubs are fully connected and each terminal is directly connected to the hub to which it is assigned. The traffic between two nodes goes from its origin to the hub of the origin, then on the direct edge to the hub of the destination and finally to its destination. In Fig. 1, an example of the considered network is shown. In the network the hub nodes are c, d, f and l. The edges of the backbone network, which is fully connected, are represented by bold lines. The routing of the traffic from terminal node e to terminal node m is represented by dashed lines. First the traffic is routed to d, to which e is assigned. Then it is routed on the direct edge from hub d to hub l, to which m is assigned, and finally it reaches m through the star tributary network.

We consider fixed costs of installing hubs and fixed costs of installing the needed capacity on each edge. The capacity needed to route the traffic on an edge is provided by the installation of an integer number of bidirectional *links* of fixed capacity. The link capacity can be different for the backbone and for the tributary edges. For each edge, the cost of establishing a link on this edge is given and this cost depends on the length of the edge. Furthermore, there is a capacity which is the maximum amount of traffic that