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## An algorithm for insertion of idle time in the single-machine scheduling problem with convex cost functions

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## Abstract

This paper addresses the problem of optimally inserting idle time into a single-machine schedule when the sequence is fixed and the cost of each job is a convex function of its completion time. We propose a pseudo-polynomial time algorithm to find a solution within some tolerance of optimality in the solution space, i.e., each completion time will belong to a small time interval z within which the optimal solution lies. Letting H be the planning horizon and |J| the number of jobs, the proposed algorithm is superior to the current best algorithm in terms of time-complexity when |J| < H/z.

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## 1. Introduction

Idle time insertion arises in just-in-time (JIT) environments where costs associated with early completion time of jobs are relevant. In the scheduling taxonomy, this kind of problem belongs to the class with nonregular measures of performance [1, p. 12], where a later job completion time can decrease the total cost associated to the schedule.

The majority of scheduling models in JIT environments considers linear objective functions, hereafter objectives. Nevertheless, a model with convex objective can often improve the quality of the representation of the real problem without greatly increasing the complexity of its solution.

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There are many industrial settings where a convex function can significantly improve the representation of the problem. A first example can be cited in the delivery of perishables such as fruits, bakery, or dairy products to the retailer where earliness costs grow more than proportionally to the earliness. Another example relates to the goodwill costs associated with the delivery of medicines to pharmacies and wholesalers. Safety inventories generally absorb a certain level of tardiness, but from the patient's point of view, the costs incurred may be huge.

Idle time insertion has been extensively studied over the past 20 years, during which time solutions considering the general case of convex functions have been scarce. In this paper we adapt for the convex case an existing algorithm that treats linear objectives, generalizing previous results. In a recent survey, Kanet and Sridharan [2, p. 106] suggest the treatment given here (the use of search methods) as an open avenue for the further development of Algorithms for Insertion of Idle Time (AIIT). Alternatively, an AIIT can be seen as a procedure, which given a job permutation, finds a semi-active schedule for the problem under analysis. For the definition of semi-active schedules and a more extensive treatment of this subject, see Davis and Kanet [3].

This work is based on Garey et al. [4] who proposed an AIIT for the earliness-tardiness problem and many definitions are given here for reference. The algorithm presented here is non-exact with time-complexity not dominated by the current best algorithm.

Let  $J = \{1, 2, ...\}$  be a finite ordered set of indices of all jobs. Every job  $j(j \in J)$  has a specified processing time  $p_j$ , a due date  $d_j$ , and a convex function  $g_j(C_j)$  that defines the cost incurred in the completion time  $(C_j)$  of the job.

The work accomplished considers that a sequence has been defined previously by some other appropriate procedure to that end. The schedule defines the assignment of the completion time  $C_j$  for each job in such way that two jobs cannot be processed at the same time, except (for the sake of simplicity) at the endpoints of the processing intervals where an overlap can exist. For a particular job *j*, there is a processing interval defined as  $[C_j - p_j, C_j]$  where  $[\cdots]$  represents a closed interval. More formally, we define the schedule  $\pi$  of a set of jobs *J* as the assignment of the completion times of all jobs, that is,  $\pi = \{C_1, C_2, \ldots, C_{|J|}\}$ , where  $C_j = C_{j-1} + W_j + p_j$ ,  $C_0 = 0$  and  $W_j$  is the idle time before starting *j*. A partial schedule  $\pi_j$  is a schedule that contains the first *j* jobs of *J*, and accordingly, a complete schedule  $\pi_{|J|} = \pi$  is a schedule that contains all jobs of *J*.

The schedule is accomplished by trying to finish the jobs at their due dates  $d_j$ , taking into account that every job requires a certain amount of time for its processing. The objective studied in this paper can be defined as min  $g(\pi) = \sum_{j \in J} g_j(C_j)$ .

The remainder of the paper is organized as follows: Section 2 describes the literature related to this work. In Section 3 we describe the proposed algorithm and the rationale underlying it. Section 4 gives some details of the computational questions related to the algorithm. Some interesting remarks about the object of this work are discussed in Section 5.

## 2. Literature review

The insertion of idle time makes sense only when a later completion time of the job can decrease the total cost of the schedule. The literature related to the insertion of idle time is always associated with problems containing earliness and tardiness. The tardiness of a job j is defined as

2286

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