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REVIEW

Convexity-preserving Bernstein–Bézier quartic scheme

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Abstract A C^1 convex surface data interpolation scheme is presented to preserve the shape of scattered data arranged over a triangular grid. Bernstein–Bézier quartic function is used for interpolation. Lower bound of the boundary and inner Bézier ordinates is determined to guarantee convexity of surface. The developed scheme is flexible and involves more relaxed constraints.

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Contents

1. Introduction

Shape preserving scattered data interpolation is always desirable in geometric modeling, visualization, engineering, sectional drawing, designing pipe systems in chemical plants, surgery, designing car bodies, ship hulls and airplane, geology, meteorology, etc. In general, ordinary interpolating techniques do not preserve the shapes of data. In this paper, we have developed a method to preserve the shape of scattered data when it is convex. Convexity is an important shape property and its applications are in designing of telecommunication system, nonlinear programming, engineering, optimization theory, parameter estimation, approximation theory [1–3].

In recent years, a good amount of work has been published on the shape preservation of univariate and bivariate convex data. It is very hard to detail all these existing schemes. Some of the noticeable contributions are reviewed here. Cai [4] presented a four-point ternary subdivision scheme for the convexity preservation of curve data. The parameters were constrained to preserve the convexity and the generated limit curve was C^2. Levin and Nadler [5] introduced a one parameter family of C^∞ algebraic curves and discussed its properties. The convexity-preserving scheme was developed using these curves to create a convex curve from a convex polygon. The method was further generalized to the convex-preserving C^∞ interpolation in R^3 by algebraic surfaces. Pan and Wang [6] proposed an automatic parametric convexity-preserving scheme for curve data. A family of interpolating spline curves with a shape parameter was introduced. The range of these parameters was determined for the convexity preservation of global and piecewise convex data points. Yong-juan and Guo-jin [7] developed a new trigonometric polynomial curve with a shape parameter for the convexity preservation of convex curve data. The trigonometric polynomial curve was obtained by the blending of parameterized polygon and trigonometric polynomial splines. This construction resulted in the automatic generation of trigonometric polynomial curves with $C^2(G^2)$ continuity. The range of the shape parameter was determined for the convexity preservation of curve data. Floater [8] defined convexity and rational convexity preservation of systems of functions and proved that total positivity and rational convexity preservation are equivalent. Roulier [9] introduced a data refinement scheme to preserve the shape of convex data arranged over the rectangular grid. The refined bivariate data could be interpolated by any standard surface interpolation technique. Iqbal [10] modified the bivariate interpolation scheme [9] and developed more relaxed constraints.

Lai [11] derived some sufficient conditions on the B-net of a multivariate Bernstein–Bézier polynomial to preserve the shape of convex data. In [11], author also discussed the sufficient conditions for the convexity of bivariate box spline surfaces. Lai [2] used bivariate C^1 cubic splines to preserve the shape of convex scattered data. In [2], convexity preserving interpolation problem was set as quadratically constrained quadratic programming problem. Quadratic programing problem was simplified to linearly constrained quadratic programming problem. Piah et al. [3] constructed a bivariate C^1 interpolant to preserve the shape of convex scattered data. The surfaces are comprised of cubic Bézier triangular patches and the sufficient conditions of convexity were derived as lower bounds of Bézier points. In a triangular patch where convexity is lost, the initial gradients at the data sites are modified so as to satisfy the sufficient conditions for convexity. Renka [12] developed a Fortran 77 software package for constructing a C^1 convex surface that interpolates arbitrarily distributed convex data. The set of nodal gradients were modified to make a convex surface from the convex nodal values and gradients. Schumaker and Speleers [13] constructed the sets of adequate linear conditions to ensure convexity of a triangle by Bernstein–Bézier method.

The study of this paper has proposed a C^1 convex scattered data interpolation scheme using Bernstein–Bézier quartic function. The Bernstein–Bézier quartic function has three inner, nine boundary and three vertex ordinates. The lower bounds of the inner and boundary Bézier ordinates are determined to preserve the convex shape of data. Since the Bernstein–Bézier quartic function has five more control points(Bézier ordinates) than cubic function [3], the convexity-preserving Bernstein–Bézier quartic scheme of this paper more accurately follows the convex shape of data as compared to [11]. In [2], the sufficient conditions for the convexity preservation of scattered data were in the form of system of inequalities with Bézier ordinates as parameters. The convexity preserving scheme of this papers has a unique lower bound for all the Bézier ordinates; thus, it is simple in implementation as compared to [11]. In [2], the convexity-preserving problem was transformed to a quadratic programming problem; thus, it is computationally expansive than the proposed convexity-preserving Bernstein–Bézier quartic scheme. The authors in [11] and [2] did not provide any numerical example of the developed convexity-preserving scheme.

The remainder of the paper is organized as follows: In Section 2, the Bernstein–Bézier quartic function [14] is rewritten. In Section 3, constraints are also derived on the Bézier