European Journal of Operational Research 246 (2015) 421–434

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

Bi-Objective Multi-Mode Project Scheduling Under Risk Aversion

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ARTICLE INFO

Article history:
Received 5 June 2014
Accepted 3 May 2015
Available online 9 May 2015

Keywords:
Project scheduling
Multi-objective optimization
Stochastic optimization
Risk aversion
Stochastic dominance

ABSTRACT

The paper proposes a model for stochastic multi-mode resource-constrained project scheduling under risk aversion with the two objectives makespan and cost. Activity durations and costs are assumed as uncertain and modeled as random variables. For the scheduling part of the decision problem, the class of early-start policies is considered. In addition to the schedule, the assignment of execution modes to activities has to be selected. To take risk aversion into account, the approach of optimization under multivariate stochastic dominance constraints, recently developed in other fields, is adopted. For the resulting bi-objective stochastic integer programming problem, the Pareto frontier is determined by means of an exact solution method, incorporating a branch-and-bound technique based on the forbidden set branching scheme from stochastic project scheduling. Randomly generated test instances, partially derived from a test case from the PSPLIB, are used to show the computational feasibility of the approach.

1. Introduction

The Resource-Constrained Project Scheduling Problem (RCPSP) has turned out as a useful and flexible modeling approach in several branches of project management (see, e.g., Brucker, Drexl, Möhring, Neumann, and Pesch, 1999). For situations where projects have to be carried out under limited resources, the RCPSP model allows the determination of optimal schedules with respect to an objective such as the makespan, i.e., the total project completion time. Despite the RCPSP's large versatility, however, there are aspects of real-life project scheduling that are not covered by the classical formulation of this problem, which has led to different extended versions. Three of the most important aspects of this kind are: (i) the necessity to take *uncertain* activity durations into account, (ii) the case where for single activities of a project, there is a choice between different *modes*, and (iii) the occurrence of further objectives in addition to the makespan, for example, cost (or profit) not considered as a constraint, but as another objective.

A large number of articles (cf. the surveys Hartmann and Briskorn, 2010; Herroelen and Leus, 2005) have dealt with these three complicating aspects in separation from each other, which has led to (i) stochastic, (ii) multi-mode or (iii) multi-objective generalizations of the RCPSP, respectively. However, a work addressing all three aspects in a *combined* model seems to be missing. To develop such a model and a suitable solution technique is the first aim of the present paper.

The second aim concerns the aspect of risk aversion. The most common approaches of stochastic project scheduling suppose a risk-neutral project manager, i.e., a decision maker optimizing expected values of outcomes. However, in the practice of project management, this is often not sufficient: special care needs to be taken to avoid extreme values of time and cost, even if their occurrence has a comparably small probability. To represent risk aversion in our optimization model, we shall resort to an approach recently developed in other fields, namely stochastic optimization under multivariate dominance constraints (see Dentcheva & Ruszczyński, 2009). This method fixes a reference solution (e.g., a state-of-the-art solution) and solves then an optimization problem on the constraint that the solution to be proposed stochastically dominates the reference solution. Contrary to the standard applications of this method, the optimization problem under consideration will be *bi-objective* in our case. Its entire efficient frontier – including so-called non-supported solutions – will be determined.

In total, we shall formulate a stochastic multi-mode RCPSP under risk aversion with the two objectives makespan and cost, and compute its Pareto frontier by means of an exact optimization method. The proposed method will make use of (i) an algorithm for optimization under multivariate dominance constraints, developed by Homem-de Mello and Mehrotra (2009), and of (ii) a problem-specific branch-and-bound technique for stochastic project scheduling investigated by Stork (2001). It will be shown that by a suitable modification and integration of these algorithms and by putting them into the context of the well-known epsilon-constraint method for multi-objective optimization, instances of nontrivial size of the considered complex problem can still be solved to optimality.

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http://dx.doi.org/10.1016/j.ejor.2015.05.004
0377-2217/

The paper is organized as follows. Section 2 gives a survey on related work and on the features distinguishing our approach from the available literature. Section 3 formulates the investigated bi-objective stochastic optimization model in mathematical terms, and Section 4 presents the suggested solution algorithms. In Section 5, computational results are presented. Section 6 contains concluding remarks.

2. Related literature

Methods of *stochastic project scheduling* have been intensely investigated in the literature, starting with works by Radermacher, Igelmund, Möhring, Stork and others (see, e.g., Igelmund and Radermacher, 1983; Möhring and Stork, 2000; Stork, 2001). Contrary to deterministic project scheduling problems, where a solution is described by a vector of starting times of activities, a solution to a stochastic project scheduling problem is a (dynamic) *policy*. It turns out that optimization on the class of *all* dynamic scheduling policies for a given stochastic scheduling problem is usually not a viable approach, partly because of the computational complexity of finding the optimal policy, partly because of the difficulties related to the description and practical implementation of a general dynamic policy. Therefore, different subclasses of policies have been proposed. Among them, let us mention the class of *early-start policies* (ES policies), the class of *activity-based priority policies*, and the class of *resource-based priority policies*. A thorough classification and description of project scheduling policies can be found in Stork (2001). In the present paper, we shall choose the framework of ES policies.

Recent research in stochastic project scheduling focuses on several issues, some of which will be outlined in the following. Ballestin (2007) discusses the circumstances under which it is advisable to use stochastic instead of deterministic scheduling methods and develops algorithmic solution techniques. Zhu, Bard, and Yu (2007) address the problem of setting target due dates by using a two-stage stochastic programming framework. Tereso, Araujo, Moutinho, and Elmaghraby (2008) investigate dynamic programming as well as diverse metaheuristic algorithms on a special stochastic resource allocation problem in project scheduling.

Ballestin and Leus (2009) present a comprehensive investigation of the RCPSP under stochastic activity durations, considering diverse objective functions such as the expected makespan, the makespan standard deviation and the probability of meeting a due date. As the computational solution technique, a greedy randomized adaptive search procedure (GRASP) is applied. Their study is of special relevance for the topic of the current paper, since they explicitly address the issue of risk aversion. From the numerical results on correlations between the different objective function values, they conclude that it suffices to focus on the expected makespan (which would be a risk-neutral consideration), since the risk-related measures are strongly correlated with the expected makespan anyway. Let us emphasize, however, that the investigations in Ballestin and Leus (2009) refer to the *single-mode* RCPSP, and that "cost" is not taken into account as an *objective*. In the present paper, we will turn to the multi-mode RCPSP, representing the total cost of the activities in the chosen modes as a separate objective function. This leads to a natural tradeoff not only between makespan and cost, but (as we shall argue in Section 3.4) also between low-risk, medium-expected-effort solutions on the one hand and high-risk, low-expected-effort solutions on the other hand. In such a context, the decisions preferred by a risk-averse decision maker may be completely different from those that are optimal under a risk-neutral stance.

Ashtiani, Leus, and Aryanezhad (2011) show that by introducing a preprocessing phase where some sequencing decisions are already made *a priori* while the remaining decisions are made dynamically, the performance of solution algorithms for the RCPSP with stochastic activity durations can be considerably improved. Deblaere, Demeulemeester, and Herroelen (2011a) combine the deployment of a scheduling policy with the determination of a vector of predictive starting times of the single activities. A "policy execution cost" composed of expected penalties for earliness and tardiness of the single activities is minimized.

Leus and Herroelen (2004) and Leus (2011) focus on ES policies and use resource flow variables to represent resource-allocation decisions, which allows the derivation of theoretical results on optimal ES policies. Furthermore, in Leus (2011), an alternative to the "forbidden set branching scheme" from Stork (2001) for enumerating feasible ES policies is proposed by the introduction of a binary branching strategy. Artigues, Leus, and Nobibon (2013) turn from stochastic optimization to robust optimization by minimizing, instead of the expected makespan, the maximum absolute regret over a set of scenarios for the durations of the activities. The two last-mentioned articles are especially interesting in the context of the present paper since we rely here on ES policies as well. Furthermore, let us note that also Artigues et al. (2013) implicitly presupposes a risk-averse attitude of the decision maker, but the model in Artigues et al. (2013) puts less emphasis on the "average situation" than the model presented here where the goal of avoiding too risky decisions is balanced with that of keeping the *expected* makespan low.

In the absence of resource constraints, *multi-mode* versions of the project scheduling problem have been investigated under the terms *time-cost tradeoff problem* or *activity crashing problem*, a problem class for which also stochastic variants exist (see, e.g., Gutjahr, Strauss, and Wagner, 2000). For the resource-constrained deterministic problem variant, a wealth of literature is available. Most but not all of these articles rely on heuristic solution techniques (see, e.g., Bouleimen and Lecocq, 2003; Coelho and Vanhoucke, 2011; Deblaere, Demeulemeester, and Herroelen, 2011b; Jozefowska, Mika, Rozycki, Waligora, and Weglarz, 2001). Much less well-investigated is the stochastic multi-mode problem variant: only very few articles dealing with this subject can be found. Tereso, Araujo, and Elmaghraby (2004) and Tereso et al. (2008) treat the resource allocations to the activities as decision variables; different resource allocations incur different costs. The "work contents" of the activities are considered as random variables. Chen, Zhang, Liu, and Liu (2010) consider a stochastic project scheduling problem with uncertain activity durations where each activity i is associated with a set M_i of execution modes. A solution is given by a permutation of the activities and a vector of execution modes. A serial schedule generation scheme is used to compute a concrete schedule for the activities, given a solution and a realization of the random variables. As the objective function, the expected net present value of the resulting cash flows is chosen. The model is solved by means of the ant colony optimization metaheuristic. The present paper has some features in common with Chen et al. (2010), but differs from Chen et al. (2010) by (i) considering makespan and cost as two separate objective functions whose Pareto frontier is to be determined, by (ii) replacing the (risk-neutral) expected-value consideration by a risk-averse consideration, and (iii) by solving the resulting problem exactly instead of heuristically.

Also Muller (2011) presents a multi-mode stochastic project scheduling problem. In their formulation, however, also *non-renewable* resources are considered, and uncertainty does not reside in the duration of the activities, but instead in the non-renewable resource requirements of each mode. In the model, the stochastic aspect is dealt with by a chance constraint. A branch-and-cut technique is used for the computational solution of the resulting conic quadratic integer program. As in Chen et al. (2010), the model is single-objective.

Godinho and Branco (2012) study a multi-mode stochastic scheduling problem with a weighted sum of expected cost and expected tardiness as the objective function. Compared to Chen et al. (2010), their model does not consider resource constraints. On the other hand, it provides additional flexibility in the considered scheduling policies insofar as the execution modes need not to be