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Commodity derivatives pricing with cointegration and stochastic covariances

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ABSTRACT

Empirically, cointegration and stochastic covariances, including stochastic volatilities, are statistically significant for commodity prices and energy products. To capture such market phenomena, we develop a continuous-time dynamics of cointegrated assets with a stochastic covariance matrix and derive the joint characteristic function of asset returns in closed-form. The proposed model offers an endogenous explanation for the stochastic mean-reverting convenience yield. The time series of spot and futures prices of WTI crude oil and gasoline shows cointegration relationship under both physical and risk-neutral measures. The proposed model also allows us to fit the observed term structure of futures prices and calibrate the market-implied cointegration relationship. We apply it to value options on a single commodity and on multiple commodities.

1. Introduction

Commodity derivative market had tremendous growth in recent years. A number of stylized facts about commodity prices have been uncovered in the literature. Although Bessembinder, Coughenour, Seguin, and Smoller (1995) and Schwartz (1997) observe the mean reversion of commodity spot prices, other researchers have identified a long-term equilibrium relationship between multiple commodities, termed cointegration. In addition, Trolle and Schwartz (2009) recognize the unspanned stochastic volatility in commodity markets, particularly that in the crude oil market. Cointegration and stochastic volatility are important ingredients in derivatives pricing and hedging. Wong and Lo (2009) investigate the valuation of options on a single underlying asset following a mean-reverting lognormal process with stochastic volatility. Duan and Pliska (2004) consider the valuation of crack spread options on two cointegrated assets. They employ an error correction model with GARCH to value the spread option, but assume the correlation between the two assets to be a constant value. Dempster, Medova, and Tang (2008) model the spread process of cointegrated assets directly using two latent factors, as the correlation between asset returns is notoriously difficult to model. Marroquín-Martínez and Moreno (2013) and Rambeerich, Tangman, Lollchund, and Bhuruth (2013) discuss computational methods for option valuation under multifactor models.

Not only are volatilities stochastic, but the correlations between pairs of assets in the energy market are also highly stochastic (see, e.g., Alexander, 1999; Kouvelis, Li, & Ding, 2013). Thus assuming a constant correlation structure is inappropriate. In this paper, we propose a tractable continuous-time model that simultaneously captures cointegration, stochastic volatilities, and stochastic correlations. More specifically, the cointegration structure is reflected by the diffusion limit of the discrete error correction model consistent with Duan and Pliska (2004), Chiu and Wong (2011) and Chiu and Wong (2012). Meanwhile, the stochastic covariance matrix is assumed to follow the continuous-time Wishart autoregressive (WAR) process proposed by Bru (1991). Gourieroux, Jasiak, and Sufana (2009) provide a thorough analysis of the properties of WAR processes in both discrete and continuous-time settings. Using the WAR process to model the covariance matrix is appealing as it does not require additional constraints to ensure the positivity or symmetry of the matrix. Buraschi, Porchia, and Trojani (2010) investigate stochastic correlation risk in asset allocation using a WAR process. Chiu and Wong (2014) generalize it to the mean-variance portfolio objective. Gourieroux and Sufana (2010) exploit the use of WAR processes for multivariate stochastic volatilities. Under the proposed model, we derive the closed-form solutions for the joint characteristic function of asset returns and futures contracts.

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The convenience yield, defined as the flow of benefit of immediate ownership of a physical commodity, is regarded as a distinguished feature of commodities as an asset class. Casassus and Collin-Dufresne (2005) document that mis-specification of the convenience yield can have a significant impact on option valuation and risk management. Therefore, correctly modeling the convenience yield process is a significant step towards modeling commodity-related contingent claims. We here document the stylized fact regarding the dynamics of the commodity convenience yield predicted by the proposed model and the traditional theory of storage, respectively. One insight of our framework is that the proposed model can be interpreted as arbitrage-free models of commodity spot prices, where the convenience yield is of stochastic volatility. To our knowledge, the implications of cointegration to stochastic convenience yield have never been explored in the literature. Using daily data on WTI crude oil and gasoline, we perform three regression tests on the market-implied convenience yield, to explore the economical and statistical significance of our model specification in commodity market. Although our empirical results reconfirm the theory of storage in Routledge, Seppi, and Spatt (2000), it also strongly support the hypothesis that cointegration is an important contributor to convenience yields. The overall explanatory power of our proposed model is much stronger than that of the traditional theory of storage. It implies that commodity market anticipates cointegration relationship under both physical and risk-neutral measures.

In the literature, Hilliard and Reis (1998) and Casassus and Collin-Dufresne (2005) assume exogenous mean-reverting convenience yield model to improve the valuation of commodity derivatives. Under the proposed model, cointegrating factors form a mean-reverting system and are contributors to convenience yield. Therefore, our model offers an endogenous explanation for the stochastic mean-reverting convenience yield. In addition, as the cointegrating factor is a linear combination of log-spot-prices, and spot prices exhibit stochastic correlations and volatilities, our model also predicts that the volatility of the convenience yield is stochastic. This prediction coincides exactly with Broadie, Detemple, Ghysels, and Torres (2000), who use a non-parametric approach to show that putting the stochastic dividend and volatility together improves empirical option prediction. Furthermore, the proposed model allows the correlation structure between spot prices and convenience yields be time-varying, as desired by Routledge et al. (2000).

The proposed model is also appealing in terms of calibration procedures as it can exactly fit to the market-observed term structure of futures prices. We further develop a way to individually filter out the risk-neutral cointegration relationship by examining the elasticity of futures prices to spot prices. The calibrated characteristic function can be applied to the valuation of options on a single commodity and on multiple commodities using fast Fourier transform (FFT) techniques.

The remainder of this paper is organized as follows. Section 2 presents the proposed model under the physical measure and derives the joint characteristic function of asset returns. Section 3 examines the stylized facts of convenience yield process predicted by the proposed model and performs comprehensive empirical analysis using crude oil and gasoline data to demonstrate the economical and statistical significance of our model specification. Section 4 discusses the calibration of the model and applies it to the pricing of commodity derivatives. Section 5 concludes the paper.

2. The model

This section introduces and studies a continuous-time dynamics of cointegrated assets with stochastic covariance. To better understand our model and its implications, we start reviewing the two-asset case of the continuous-time cointegration model of Phillips (1991) and Duan and Pliska (2004). A stochastic covariance matrix is further introduced to simultaneously capture cointegration and stochastic covariances, which enables explicit formulation of joint characteristic function. We also discuss the corresponding risk-neutral asset dynamics and closed-form pricing of futures contract.

2.1. Review of the continuous-time cointegration model

We start with the spot-price process of two risky assets, $S_1(t)$ and $S_2(t)$, with a constant covariance matrix. Let $X_j(t) = \ln S_j(t)$ for $j = 1, 2$. The continuous-time cointegration model of Phillips (1991) and Duan and Pliska (2004) under the physical probability measure is given by

$$\begin{aligned} dX_1(t) &= \left[\theta_1 - \kappa_{11}X_1(t) - \kappa_{12}X_2(t) + \lambda_1\sigma_1^2\right]dt + \sigma_1\, d\widetilde{Z}_1 \\ dX_2(t) &= \left[\theta_2 - \kappa_{21}X_1(t) - \kappa_{22}X_2(t) + \lambda_2\sigma_2^2\right]dt + \sigma_2\, d\widetilde{Z}_2, \end{aligned} \quad (1)$$

where $\widetilde{Z}_1$ and $\widetilde{Z}_2$ are correlated Wiener processes with correlation coefficient $\widetilde{\rho}$, and other parameters, i.e., θ_i, κ_{ij}, λ_i, and σ_i for $i, j = 1, 2$, are constant values. In (1), λ_1 and λ_2 are market prices of risks. θ_1 and θ_2 are the equilibrium mean level of the log-asset values. If $\kappa_{12} = \kappa_{21} = 0$, X_1 and X_2 are mean-reverting assets individually. Let κ be the 2×2 matrix collecting κ_{ij}, for $i, j = 1, 2$. (1) is said to be a cointegration system if two eigenvalues of κ are non-negative and at least one of them is positive. If $\vec{e}$ denotes an eigenvector associated with a positive eigenvalue of κ, $[X_1(t)X_2(t)] \cdot \vec{e}$ is a cointegrating factor of the system (1). The covariance matrix between $X_1(t)$ and $X_2(t)$ is constant and deduced to be

$$V = \begin{pmatrix} \sigma_1^2 & \widetilde{\rho}\sigma_1\sigma_2 \\ \widetilde{\rho}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}.$$

However, such a model is unrealistic for certain circumstances. For example, Alexander (1999) pointed out that the correlation between pairs of assets in the energy market is highly stochastic. In this paper, we propose to model the covariance matrix V as a stochastic matrix following the continuous-time Wishart autoregressive process.

2.2. The proposed model of cointegration and stochastic covariances

Consider n risky assets whose prices at time t are represented in an n-dimensional vector S_t. The $n \times n$ covariance matrix at time t denoted by V_t is stochastic and positive definite. We postulate the joint dynamics of logarithmic spot prices $X_t = \ln S_t$ and V_t under the physical measure $\mathbb{P}$ as

$$dX_t = [\theta(t) - \kappa X_t + (\mathrm{Tr}(D_1V_t), \ldots, \mathrm{Tr}(D_nV_t))']dt + \sqrt{V_t}dZ_t^{\mathbb{P}}, \quad (2)$$

$$dV_t = (\beta Q'Q + MV_t + V_tM')dt + \sqrt{V_t}dW_t^{\mathbb{P}}Q + Q'\left(dW_t^{\mathbb{P}}\right)'\sqrt{V_t}, \quad (3)$$

where $Z_t^{\mathbb{P}} \in \mathbb{R}^n$ and $W_t^{\mathbb{P}} \in \mathbb{R}^{n\times n}$ are a vector Brownian motion and a matrix Brownian motion under the physical measure $\mathbb{P}$, respectively. The vector $\theta(t)$ represents the equilibrium mean level of the log-asset values against time. κ is an $n \times n$ constant matrix of cointegration coefficients. If κ is a positive diagonal matrix, all of the individual assets are stationary and exhibit *mean-reversion*. β is a scalar, such that $\beta > n - 1$, and M and Q are $n \times n$ constant real square matrices. Tr $(\cdot)$ denotes the trace operator. Q' is the (unconjugated) transpose of matrix Q. $\sqrt{V_t}$ is the positive symmetric square root of V_t such that $\sqrt{V_t}\sqrt{V_t} = V_t$. The continuous-time process of the stochastic covariance matrix in (3) is essentially the continuous-time Wishart autoregressive (WAR) process introduced by Bru (1991). The WAR process ensures that, at any point in time, the stochastic covariance matrix is positive definite. $\mathrm{Tr}(D_iV_t)$ in (2) represents a risk premium of V_t. If Q, M, and $W_t^{\mathbb{P}}$ are all diagonal matrices, V_t is also a diagonal matrix with each of its diagonal elements following the stochastic volatility model of Heston (1993) and therefore our model embraces the model of Wong and Lo (2009) when $n = 1$.