European Journal of Operational Research 246 (2015) 554–561

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

A nonparametric methodology for evaluating convergence in a multi-input multi-output setting

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ARTICLE INFO

Article history:
Received 25 May 2014
Accepted 4 May 2015
Available online 12 May 2015

Keywords:
Convergence
Productivity
Malmquist index
Data envelopment analysis
Construction industry

ABSTRACT

This paper presents a novel nonparametric methodology to evaluate convergence in an industry, considering a multi-input multi-output setting for the assessment of total factor productivity. In particular, we develop two new indexes to evaluate σ-convergence and β-convergence that can be computed using nonparametric techniques such as Data Envelopment Analysis. The methodology developed is particularly useful to enhance productivity assessments based on the Malmquist index. The methodology is applied to a real world context, consisting of a sample of Portuguese construction companies that operated in the sector between 2008 and 2010. The empirical results show that Portuguese companies tended to converge, both in the sense of σ and β, in all construction activity segments in the aftermath of the financial crisis.

1. Introduction

Convergence has been extensively discussed in the economic growth literature over the past decades (see Islam, 2003; Temple, 1999 for a literature review). The first studies devoted their attention to whether poor or low productive countries would catch up with their rich or highly productive counterparts. This is an issue of utmost importance for researchers and policymakers interested in worldwide welfare.

Two main concepts of convergence appear in the classical literature (see Barro & Sala-i Martin, 1992). β-convergence analyzes if poor countries tend to grow faster than rich countries, whereas σ-convergence examines if the dispersion of the productivity for a group of countries tends to decrease over time. To measure these concepts, most studies use a single productivity measure, such as income per capita or Gross Domestic Product (GDP) per capita, or a measure of total factor productivity estimated using econometric methods. The evaluation of convergence in a multi-input multi-output setting has not been addressed in the literature.

This research contributes to the literature by developing a nonparametric methodology for the evaluation of convergence in a multi-input multi-output setting, to enhance productivity assessments based on the Malmquist index. In particular, we explore the use of Shephard distance functions, estimated through Data Envelopment Analysis (DEA), to calculate β-convergence and σ-convergence. In addition, this paper applies the two new indexes to evaluate convergence in a sample of Portuguese construction companies that operated in the sector between 2008 and 2010.

The remainder of this paper is organized as follows. Section 2 describes the new nonparametric indexes to evaluate σ-convergence and β-convergence, and explains their computation using DEA. Section 3 presents the empirical application, including the motivation, the description of the data set and the discussion of the results. The last section concludes and points topics for future research.

2. Evaluation of convergence using nonparametric techniques

2.1. A new nonparametric σ-convergence index

The concept of σ-convergence can be explained as follows: "a group of economies are converging in the sense of σ if the dispersion of the their real per capita GDP levels tends to decrease over time" (Sala-i Martin, 1996). The notion of σ-convergence can be expressed, in mathematical terms, as shown in expression (1).

$$\sigma_{t+1} < \sigma_t \quad (1)$$

σ_t is the standard deviation of the logarithm of the real per capita GDP levels across all economies in period t, whereas σ_{t+1} is a similar measure in a subsequent period. This definition can be extended to a more general setting, consisting of the assessment of Decision Making Units (DMUs) (e.g., representing countries or organizations), where σ-convergence explores whether the dispersion of values of a productivity indicator, measured as an output to input ratio, tends to decrease (or increase) over time.

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http://dx.doi.org/10.1016/j.ejor.2015.05.015
0377-2217/

Our aim is to generalize the measurement of convergence to a multi-input multi-output setting, and thus we explore σ-convergence using the Shephard distance function. The Shephard distance function is a generalization of the production function that allows us to consider simultaneously multiple inputs and outputs. As this paper is devoted to output oriented assessments, all indexes and measures described have an output orientation.

Consider n DMUs in time period t that use inputs $x^t \in R_+^m$ to produce outputs $y^t \in R_+^s$, and in a subsequent time period $t+1$ use inputs $x^{t+1} \in R_+^m$ to produce outputs $y^{t+1} \in R_+^s$. In period t, the production technology T^t consists of the set of all feasible input/output combinations for a certain production process, as shown in (2).

$$T^t = \{(x^t, y^t) : x^t \text{ can produce } y^t\} \tag{2}$$

Following Shepard (1970), the output distance function for DMU j_o in relation to the technology T^t is defined as shown in (3).

$$D^t(x^t, y^t) = \min\left\{\theta : \left(x^t, \frac{y^t}{\theta}\right) \in T^t\right\} \tag{3}$$

This function is defined as the reciprocal to the maximum proportional expansion of the output vector y^t, given inputs x^t, i.e., $D^t(x^t, y^t) \le 1$. This means that it corresponds to the efficiency score of DMU j_o, in the sense of Farrell (1957). Thus, in order to estimate σ-convergence we can use efficiency measures. In other words, the spread of productivity levels can be estimated using the spread of efficiency measures. Efficiency is a relative measure that compares the productivity of a DMU j_o with the best productivity levels of the sample. Therefore, a sample with larger dispersion of productivity levels will also have larger dispersion of efficiency. Fare, Grosskopf, Lindgren, and Roos (1992) were the first to note that input and output distance functions could be estimated using DEA models (Charnes, Cooper, & Rhodes, 1978).

For a given DMU j_o, the ratio between the efficiency score in period $t+1$ and in t, as presented in (4) is a measure of convergence towards the best practice frontier (see Fare, Grosskopf, Norris, & Zhang, 1994). This implies that there is convergence to the frontier if efficiency increases from period t to $t+1$. Note that this ratio corresponds to the Efficiency Change ($EC^{t,t+1}$) component of the Malmquist index proposed by Fare et al. (1992).

$$\frac{D^{t+1}(x^{t+1}, y^{t+1})}{D^t(x^t, y^t)} = EC^{t,t+1} \tag{4}$$

To determine σ-convergence in an industry, we propose calculating the geometric mean of expression (4) for all DMUs in the sample, as shown in (5). To distinguish our measure of σ-convergence, defined using Shephard distance functions and calculated using DEA models, from the traditional σ-convergence measure (see expression 1), we refer to it as $\hat{\sigma}$-convergence hereafter.

$$\hat{\sigma}\text{-convergence} = \left(\prod_{j=1}^{n} EC_j^{t,t+1}\right)^{1/n} \tag{5}$$

The $\hat{\sigma}$-convergence index may be greater, equal or smaller than one. A $\hat{\sigma}$-convergence index greater than one indicates convergence (i.e. the DMUs moved closer to the best practice frontier from period t to $t+1$), whereas a score less than one means divergence (i.e. the DMUs moved away from the best practice frontier between t to $t+1$). A score equal to one indicates that, on average, the DMUs are located at a similar distance to the frontier in period t and $t+1$.

The advantage of using $\hat{\sigma}$-convergence over the traditional measure of σ-convergence is that it allows accounting for multiple inputs and multiple outputs, as it can be estimated using nonparametric techniques such as DEA.

The basic ideas behind the calculation of the $\hat{\sigma}$-convergence can be illustrated in Fig. 1. This figure presents 10 DMUs (e.g., countries), whose activity is represented by an output (e.g., GDP) and an input (e.g., population) in two time periods. The ratio y/x represents a productivity measure. The best practice frontiers (BF) at time periods $t+1$ and t are also plotted in Fig. 1.

Consider DMU H, represented by point H^t in time period t and point H^{t+1} in time period $t+1$. In the example, the efficiency score of DMU H in period t measured in relation to the best practice in t, corresponds to the ratio oH^t/oa. The efficiency score of DMU H in period $t+1$, measured in relation to the best practice frontier in period $t+1$, corresponds to the ratio $o'H^{t+1}/o'd$. Hence, the change in efficiency between period t and $t+1$ is measured as shown in expression (6).

$$EC^{t,t+1} = \frac{o'H^{t+1}/o'd}{oH^t/oa} \tag{6}$$

The value of $EC^{t,t+1}$ for DMU H is greater than one (i.e., 1.09), as DMU H is closer to the best practice frontier in $t+1$ than in t. This indicates convergence towards the best practice frontier.

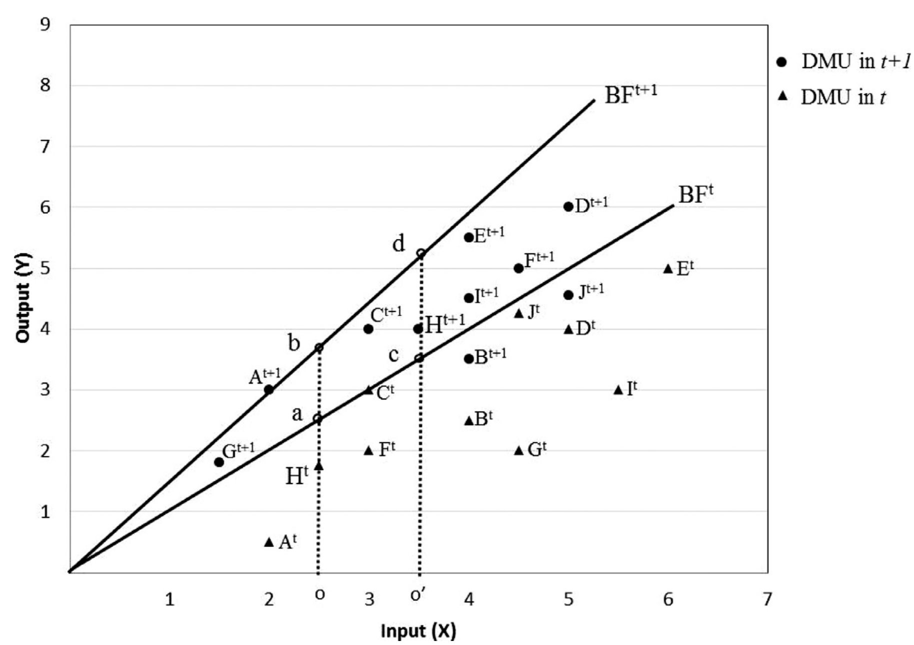

Fig. 1. Illustrative example one input–output.