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Decision Support

A stochastic dynamic pricing model for the multiclass problems in the airline industry

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ABSTRACT

In the airline industry, deciding the ticket price for each flight directly affects the number of people that in the future will try to buy a ticket. Depending on the willingness-to-pay of the customers the flight might take off with empty seats or seats sold at a lower price. Therefore, based on the behavior of the customers, a price must be fixed for each type of product in each period. We propose a stochastic dynamic pricing model to solve this problem, applying phase type distributions and renewal processes to model the inter-arrival time between two customers that book a ticket and the probability that a customer buys a ticket. We test this model in a real-world case where as a result the revenue is increased on average by 31 percent.

1. Introduction

In the airline industry, deciding the price for a product (i.e., the ticket) is one of the major problems to be solved on a daily basis (Talluri & Van Ryzin, 2004). On the other hand, the price that is decided for a product directly affects its future demand (Phillips, 2005). In a single flight there are different products with different demand behaviors, and the price decision is to be made for each one of these products. Also, the product offered by the airlines cannot be replenished and is perishable, indicating that there is a finite number of products and a limited time period to which the decision is confined. If at the beginning of a period a low price is offered for a particular product then it is more likely that many potential customers buy the tickets at a lower price than what they would be willing to pay. On the other hand if a higher price is fixed only a few potential customers are likely to buy tickets and even lowering the price at the end of the sales period might still leave some unsold tickets. Therefore, the challenge in the airline industry is to fix a price in each period of time that minimizes this loss of revenue and takes advantage of the willingness-to-pay of the customers. Because the decision taken in a particular moment of time affects what could happen in the following periods and it should also be renewed dynamically to maximize the revenue, this problem is commonly known as *dynamic pricing* (Talluri & Van Ryzin, 2004).

Revenue Management (RM) techniques have been applied to the dynamic pricing problems in the airline industry since approximately 40 years ago. The objective of applying these techniques is to dynamically change the prices during the selling period, such that each seat is sold at the maximum price possible. The prices fixed by the airline for a product during the selling periods depend on the type of product, the type of customer, and the distribution channel. The airlines not only decide the new prices by these factors but also study the competitors' decisions closely. Application of RM to airline ticket pricing has been proven to increment the revenue by between 2 percent and 8 percent (Li & Ji-hua, 2007). This in turn has increased the number of scientific studies on how to efficiently change the prices to maximize the total revenue of a flight (Talluri & Van Ryzin, 2004).

To maximize the total revenue, the price has to be changed in each period, based on the behavior of the demand. Also, in each period of time the decision to change the price for the next period should be made for each product. In the literature, to solve this problem, various dynamic pricing models are proposed. Maglaras and Meissner (2006) provide a thorough survey of the different approaches that use linear and nonlinear programming, stochastic and deterministic dynamic programming and statistical models, among others. In most of these approaches, similar assumptions are taken into account such as: Non-Homogeneous Poisson Process (NHPP) for the inter-arrival time distribution of bookings, absence of competition, known probability of buying a product, and the absence of effects such as no-shows, costs of overbooking and cancellations (Talluri & Van Ryzin, 2004). There are also some models that relax one or more of these assumptions. For example Li and Ji-hua (2007) propose a model with one competitor, Li and Chen (2009) present time-dependant arrival

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time distributions with multiple products, and Chen (2012) presents censored observation approaches where the demand is unknown.

Most of the models consulted in this paper share the assumption of NHPP for the inter-arrival time distribution of bookings. To accurately estimate the rates for the NHPP, the arrival process needs to follow an exponential distribution in each period of time. However, in reality this behavior could follow any distribution besides the exponential (Talluri & Van Ryzin, 2004). Therefore, a major challenge in this problem is how to accurately and efficiently estimate the behavior of the demand. This behavior can be divided into two parts, i.e., the demand inter-arrival time and the probability that a ticket is bought at a certain price.

There are different approaches on how to estimate the behavior of the customers in terms of their demand inter-arrival time distribution and their buying probability of a booked ticket. For example Chen (2012) applies Markov Decision Process (MDP) models with censored data, where censored data represent the non-buying customers due to product unavailability. Haensel and Koole (2011) apply a customer choice sets approach where the customers are grouped by their preferences. Also Fiig, Isler, Hopperstad, and Belobaba (2010) and Phillips (2005) propose multinomial approaches where they estimate the utility that the customer perceives over the available products. Although these methodologies try to estimate the probability distributions mentioned, most of them use the utility term which defines the customer's appeal toward a product. Therefore, it is difficult to estimate this value unless information on the different choices that a customer can make is available. One of the possible ways to get this information is by conducting interviews, which is time consuming, expensive, and requires a large sample size to create useful data.

In a typical flight different types of products can be sold (Phillips, 2005), where each product has a different behavior of demand. Most of the airlines use RM software to help decide the price at which to sell a ticket. But the models behind the software suggest the optimal price using different methods of estimating the buying probability and the demand inter-arrival distribution. Ernst and Kamrad (2006) argue that these models rely heavily on historical data and can be inaccurate. On the other hand there are airlines that do not have RM software and their decisions are based on experience and comparison with the performance of the competition, which do not necessarily guarantee the optimal solution. Therefore, in this paper, we propose a stochastic dynamic programming model to solve the problem of maximizing the revenue over a finite horizon, divided into various decision periods and with different products. In each decision period a price is going to be decided, from a pool of available prices, for each product depending on the number of seats available for each of these products. We work under the assumption that the demand inter-arrival time does not necessarily follow a NHPP, and the probability that a ticket is bought at a certain price is not known and has to be estimated as well. In this model we approximate the inter-arrival time distribution of demand and the buying probability using Phase Type (PH) distributions. The PH distributions are combinations of exponential distributions (phases) that can fit different types of behavior (Latouche & Ramaswami, 1999). We use various algorithms proposed in the literature to fit the distributions of demand inter-arrival time and the buying probability, depending on the data characteristics. Finally, we use renewal processes to implement the PH distributions in counting processes.

Having the PH estimates of the inter-arrival time distribution, applying the renewal processes, and the buying probability, we solve the stochastic dynamic program with backward induction for different scenarios, each of which with a different combination of parameters to test the performance of our proposed method. We change the number of seats, the type of distributions and finally test our proposed model on a case study with industry data.

The rest of the paper is organized as follows. In Section 2 we present the proposed model with a detailed explanation on how to estimate each of the distributions and introduce the dynamic pricing model. In Section 3 we introduce the PH family of distributions and discuss data fitting techniques to a PH distribution using different algorithms. In Section 4 we present some structural properties of the dynamic pricing model and prove the existence of an optimal solution. In Section 5 we present the result for the scenarios and the case study, dividing them into three parts: the fitting of the purchasing probability distribution, the fitting of demand inter-arrival time distribution, and the results of the dynamic pricing model. Finally in Section 6 we conclude the paper and state future steps for this research.

2. The proposed model

In this section we first introduce some airline specific jargon and definitions, then we define the essential components of our proposed dynamic pricing model. The first component consists of estimating the probability that a customer buys a previously booked ticket at a certain price. We refer to this component as the *ticketing probability*. The second component is the probability that j customers buy tickets for a given product on a particular flight in a period of time t, which we will call the *buying probability*. The buying probability depends on the ticketing probability and the inter-arrival time distribution of the demand. We estimate the buying probability by the number of bookings (reservations) that the airline receives. To estimate the behavior of the probability distributions closer to reality and with more flexibility, we propose to fit PH distributions to them. This fitting allows us to find a relatively accurate approximation to the behavior of the underlying distributions when they do not necessarily follow the form of any standard distribution (e.g., the multi-modality behavior) (Latouche & Ramaswami, 1999). Then we apply renewal processes to approximate the arrival process of bookings by a counting process. After estimating the probability distributions of ticketing and buying, we present the dynamic pricing model that attempts to find optimal pricing policies, given these distributions.

2.1. Definitions and notation

In this section we present the terms and definitions used in the context of airline revenue management. We define *booking* as the reservation of a seat where no money is involved. On the other hand, *ticketing* occurs when the customer pays for this booking. We estimate the demand for a particular ticket by the number of bookings received for that product. Using these two definitions, the probability distribution of ticketing is defined as the probability that a booked ticket is paid for and bought eventually.

A *product* is defined by the benefits that it offers. The most common benefits of a product include the number of miles received, the costs of changing the ticket, the type of cabin, and promotions. Each product can be offered in various *flying classes*, which we denote as c. Each of these flying classes has a price assigned, depending on the type of product that they represent. In general, the number of cabins, the number of products, and the number of classes vary depending on the airline. Some airlines offer only two classes while others can have up to 26 different classes in a single flight (Phillips, 2005). For each class c, only one type of product is offered, while many classes can offer the same product.

The tickets are up for sale since almost a year before the departure date of the flight, and this period is called the *selling period*. Because different classes can have the same product but with different prices, only one class is available per product in each period of time. Therefore, during the selling period the class offered for a particular product changes, hence changing the prices of the product according to the available class. So the price that a customer sees during the selling period is the price of the available class for that product. The objective of our proposed dynamic pricing model is to set the prices in each