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Decision Support

On the relations between ELECTRE TRI-B and ELECTRE TRI-C and on a new variant of ELECTRE TRI-B ☆

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ABSTRACT

ELECTRE TRI is a set of methods designed to sort alternatives evaluated on several criteria into ordered categories. The original method uses limiting profiles. A recently introduced method uses central profiles. We study the relations between these two methods. We do so by investigating if an ordered partition obtained with one method can also be obtained with the other method, after a suitable redefinition of the profiles. We also investigate a number of situations in which the original method using limiting profiles gives results that do not fit well our intuition. This leads us to propose a variant of ELECTRE TRI that uses limiting profiles. We show that this variant may have some advantages over the original method.

1. Introduction

This paper deals with ELECTRE TRI. It consists in a set of methods that are the most recent ones belonging to the ELECTRE family of methods (for overviews, see Figueira, Mousseau, & Roy, 2005, Figueira, Greco, Roy, & Słowiński, 2013, Roy & Bouyssou, 1993, chap. 5 and 6).

ELECTRE TRI was originally introduced in the doctoral dissertation of Wei (1992) (supervised by B. Roy) and was detailed in Roy and Bouyssou (1993, pp. 389–401). The original method is designed to sort alternatives evaluated on multiple criteria into ordered categories defined by *limiting profiles* (see Roy & Bouyssou, 1993, chap. 6, for a detailed analysis of the sorting problem formulation). This method has generated much interest. Indeed, sorting alternatives into ordered categories is a problem occurring in many real-world situations. Moreover, on a more technical level, the fact that the method only compares alternatives with a set of carefully selected limiting profiles that are linked by dominance greatly facilitates the exploitation of the outranking relation that is built. This limits the consequences of the fact that this relation is, in general, neither transitive nor complete (Bouyssou, 1996). Many techniques have been proposed for the elicitation of the parameters of this method (see Cailloux, Meyer, & Mousseau, 2012; Dias & Clímaco, 2000; Dias, Mousseau, Figueira, & Clímaco, 2002; Dias & Mousseau, 2003, 2006; Damart, Dias, & Mousseau, 2007; Leroy, Mousseau, & Pirlot, 2011; Mousseau & Słowiński, 1998; Mousseau, Słowiński, & Zielniewicz, 2000; Mousseau, Figueira, & Naux, 2001; Mousseau, Figueira, Dias, Gomes da Silva, & Clímaco, 2003; Mousseau & Dias, 2004; Mousseau, Dias, & Figueira, 2006; Ngo The & Mousseau, 2002; Zheng, Metchebon Takougang, Mousseau, & Pirlot, 2014). Most of them use mathematical programming tools to infer the parameters of the method, based on assignment examples. This method has been applied to a large variety of real world problems (see the references at the end of Section 6 in Almeida-Dias, Figueira, & Roy, 2010). It has received a fairly complete axiomatic analysis in Bouyssou and Marchant (2007a,b). In a nutshell, ELECTRE TRI can be considered as a real success story within the ELECTRE family of methods.

A recent paper (Almeida-Dias et al., 2010) introduced a new method that uses *central profiles* instead of limiting profiles (Almeida-Dias et al., 2010, use the term "characteristic reference action" instead of central profiles). This is an interesting development since it seems "intuitively" easier to elicit central rather than limiting profiles (a related paper, Almeida-Dias, Figueira, & Roy, 2012, deals with the case of *multiple* central profiles. We do not study this more general case in the present paper).

The present paper was prompted by the analysis of this new method and its comparison with the original one. After having recalled the essential elements of both methods (Section 2), we investigate two main points.

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We first study the relations between these two methods (Section 3). We do so by investigating if an ordered partition obtained with one method can also be obtained with the other method, after a suitable redefinition of the profiles. Our main conclusion is that this is not always possible. This fact should not be interpreted as a criticism of ELECTRE TRI but as the sign that the two methods that we study use, beyond surface, different principles.

We then present (Section 4) a number of situations in which the original method using limiting profiles gives results that do not fit well our intuition. These situations are mainly linked to the behavior of the method w.r.t. what we will call "strong dominance" and w.r.t. the transposition operation used by Almeida-Dias et al. (2010) to justify their proposal of two components of the method using central profiles that they recommend using conjointly. In particular, as first observed in Roy (2002), the two versions of the original method do not correspond via the application of this transposition operation: the pseudo-disjunctive version (also known as "optimistic") is not obtained from the pseudo-conjunctive version (also known as "pessimistic") via the transposition operation and vice versa. We detail this point that may explain why most of the elicitation techniques proposed so far only deal with the pseudo-conjunctive version (Zheng, 2012; Zheng et al., 2014, are exceptions) and why the axiomatic analysis conducted in Bouyssou and Marchant (2007a,b) is also limited to the pseudo-conjunctive version. This will lead us to propose a new variant of ELECTRE TRI using limiting profiles in which the two versions correspond via the transposition operation. We show that this new variant may have some advantages over the original method. A final section (Section 5) concludes with the indication of directions for future research.[3]

2. ELECTRE TRI: a brief reminder

We consider a set of alternatives A. Each alternative $a \in A$ is supposed to be evaluated on a family of n real-valued criteria, i.e., n functions $g_1, g_2, \ldots, g_n$ from A into $\mathbb{R}$. These criteria are built with respect to a property $\mathcal{P}$. This property is usually taken to be "preference" but it can also be "riskiness" or "flexibility". Let us define $N = \{1, 2, \ldots, n\}$. We suppose, w.l.o.g., that increasing the performance on any criterion increases the performance of an alternative w.r.t. the property $\mathcal{P}$. The *dominance* relation Δ is defined letting, for all $a, b \in A$, $a \Delta b$ if $g_i(a) \geq g_i(b)$, for all $i \in N$. In such a case, we say that *a dominates b*. We say that *a strictly dominates b* if $a \Delta b$ and $Not[b \Delta a]$, which we denote by $a \Delta^a b$, since Δ^a is the asymmetric part of Δ.

2.1. Construction of the outranking relation

In all the examples that follow, discordance will play no role and, on all criteria, the indifference and preference thresholds will be equal (hence, it is not restrictive to take them constant, see Roy & Vincke, 1987). In order to keep things simple, we briefly recall here how the outranking relation is built in this particular case. We refer, e.g.,to Almeida-Dias et al. (2010, Section 2) or to Roy and Bouyssou (1993, pp. 284–289) for the description of the construction of the outranking relation in the general case, i.e.,when indifference and preference thresholds may be unequal and may vary and when discordance plays a role. It is important to realize the definition of the outranking relation that we detail below, although simpler than the general definition, is a particular case of the general one. All relations that can be obtained using the formulae in this section can be obtained using the more general formulae presented in Almeida-Dias et al. (2010, Section 2) and Roy and Bouyssou (1993, pp. 284–289).

We associate with each criterion $i \in N$ a nonnegative preference threshold $p_i \geq 0$. If the value $g_i(a) - g_i(b)$ is positive but less than p_i, it is supposed that this difference is not significant, given the way g_i has been built. Hence, on this criterion, the two alternatives should be considered indifferent.

The above information is used to define, on each criterion $i \in N$, a valued relation on A, i.e.,a function from $A \times A$ into $[0, 1]$, called the partial concordance relation, such that:

$$c_i(a, b) = \begin{cases} 1 & \text{if } g_i(b) - g_i(a) \leq p_i, \\ 0 & \text{if } g_i(b) - g_i(a) > p_i, \end{cases}$$

(in the particular case studied here, the valued relations c_i can only take the values 0 or 1. In the general case, they can take any value between 0 and 1).

Each criterion $i \in N$ is assigned a non-negative weight w_i. We suppose, w.l.o.g., that weights have been normalized so that $\sum_{i=1}^{n} w_i = 1$.

The valued relations c_i are aggregated into a single valued outranking relation s letting, for all $a, b \in A$,

$$s(a, b) = \sum_{i=1}^{n} w_i c_i(a, b).$$

On the basis of the valued relation s, a binary relation S_λ on A is defined letting:

$$a \, S_\lambda \, b \iff s(a, b) \geq \lambda,$$

where $\lambda \in [0, 1]$ is a cutting level (usually taken to be above 1/2). The relation S_λ is interpreted as saying "has at least as much of property $\mathcal{P}$ as" relation between alternatives (if, as is usually assumed, property $\mathcal{P}$ is taken to be "preference", the relation S_λ is classically interpreted as an "at least as good as" relation between alternatives). From S_λ, we derive the following relations:

$$a \, P_\lambda \, b \iff [a \, S_\lambda \, b \text{ and } Not[b \, S_\lambda \, a]],$$
$$a \, I_\lambda \, b \iff [a \, S_\lambda \, b \text{ and } b \, S_\lambda \, a],$$
$$a \, J_\lambda \, b \iff [Not[a \, S_\lambda \, b] \text{ and } Not[b \, S_\lambda \, a]],$$

that are respectively interpreted as "has strictly more of property $\mathcal{P}$ as", "has as much of property $\mathcal{P}$ as", "is not comparable w.r.t. property $\mathcal{P}$ to" relations between alternatives (if property $\mathcal{P}$ is taken to be "preference", these relations are respectively interpreted as: "strictly better than", "indifferent to" and "incomparable to").

It is easy to check (Roy & Bouyssou, 1993, chap. 5) that, if $a \Delta b$, then $s(a, b) = 1$ and, for all $c \in A$, $s(b, c) \leq s(a, c)$ and $s(c, a) \leq s(c, b)$. Hence, if $a \Delta b$, we have $a S_\lambda b$ and, for all $c \in A$,

$$b \, S_\lambda \, c \Rightarrow a \, S_\lambda \, c, \quad b \, P_\lambda \, c \Rightarrow a \, P_\lambda \, c,$$
$$c \, S_\lambda \, a \Rightarrow c \, S_\lambda \, b, \quad c \, P_\lambda \, a \Rightarrow c \, P_\lambda \, b.$$

The following proposition will be useful. It is taken from Roy and Bouyssou (1993, chap. 6, pp. 392–393) and its validity is independent of the simplifying hypotheses we have made concerning the construction of the outranking relation.

Proposition 1. *Let $c^1, c^2, \ldots, c^t \in A$ be such that $c^{k+1} \Delta^a c^k$ for $k = 1, 2, \ldots, t-1$. Suppose furthermore that, for all $a \in A \setminus \{c^1, c^t\}$, $c^t \, P_\lambda \, a$ and $a \, P_\lambda \, c^1$.*

When an alternative $a \in A \setminus \{c^1, c^t\}$ is compared to the subset of alternatives $c^1, c^2, \ldots, c^t$, three distinct situations may arise:

1. $c^t \, P_\lambda \, a, \ldots, c^{k_1+1} \, P_\lambda \, a, a \, P_\lambda \, c^{k_1}, a \, P_\lambda \, c^{k_1-1}, \ldots, a \, P_\lambda \, c^1$,
2. $c^t \, P_\lambda \, a, \ldots, c^{\ell_2+1} \, P_\lambda \, a, a \, I_\lambda \, c^{\ell_2}, a \, I_\lambda \, c^{\ell_2-1}, \ldots, a \, I_\lambda \, c^{k_2+1}, a \, P_\lambda \, c^{k_2}, \ldots, a \, P_\lambda \, c^1$,
3. $c^t \, P_\lambda \, a, \ldots, c^{\ell_3+1} \, P_\lambda \, a, a \, J_\lambda \, c^{\ell_3}, a \, J_\lambda \, c^{\ell_3-1}, \ldots, a \, J_\lambda \, c^{k_3+1}, a \, P_\lambda \, c^{k_3}, \ldots, a \, P_\lambda \, c^1$,

with $t > k_1 > 0$, $t > \ell_2 > k_2 > 0$, and $t > \ell_3 > k_3 > 0$.

[3] In what follows, we will use the following terminology. ELECTRE TRI is a *set of methods*. ELECTRE TRI-B is a *method* that has two *versions*: ELECTRE TRI-B, pseudo-conjunctive, and ELECTRE TRI-B, pseudo-disjunctive. ELECTRE TRI-C is a method that has two *components*: ELECTRE TRI-C, ascending, and ELECTRE TRI-C, descending. We sometimes abbreviate ELECTRE TRI-B, ELECTRE TRI-B, pseudo-conjunctive, and ELECTRE TRI-B, pseudo-disjunctive as ETRI-B, ETRI-B-pc, and ETRI-B-pd. We also sometimes abbreviate ELECTRE TRI-C, ELECTRE TRI-C, ascending, and ELECTRE TRI-C, descending as ETRI-C, ETRI-C-a, and ETRI-C-d.