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Modeling lotsizing and scheduling problems with sequence dependent setups

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ABSTRACT

Several production environments require simultaneous planing of sizing and scheduling of sequences of production lots. Integration of sequencing decisions in lotsizing and scheduling problems has received an increased attention from the research community due to its inherent applicability to real world problems. A two-dimensional classification framework is proposed to survey and classify the main modeling approaches to integrate sequencing decisions in discrete time lotsizing and scheduling models. The Asymmetric Traveling Salesman Problem can be an important source of ideas to develop more efficient models and methods to this problem. Following this research line, we also present a new formulation for the problem using commodity flow based subtour elimination constraints. Computational experiments are conducted to assess the performance of the various models, in terms of running times and upper bounds, when solving real-word size instances.

1. Introduction

Several companies face the problem of timing and sizing production lots over a given planning horizon. Additionally, in many of these production environments, switching between production lots of two different products triggers operations, such as machine adjustments and cleansing procedures. These setup operations, which are dependent on the sequence, consume scarce production time and may cause additional costs due to, for example, losses in raw materials or intermediate products. Consequently, the production sequence must be explicitly embedded in the lot definition and scheduling. Lot sizing determines the timing and level of production to satisfy deterministic product demand over a finite planning horizon. Sequencing establishes the order in which lots are executed within a time period, accounting for the sequence-dependent setup times and costs. Integration of these two problems enables the creation of better production plans than those obtained when solving the two problems hierarchically by inducing the solution of the lotsizing problem in the scheduling level. Production plans are created with the objective of minimizing the overall costs consisting mainly of stock holding and setups, while satisfying the available capacity in each time period from which the expenditure in setup times is deducted.

This production scenario is present in many process industries, in which an efficient use of the available capacity is key to stay competitive in the current market environment. In the beverage industry sequence dependent setups occur in bottling lines when switching between two products that differ in the container size and/or container shape and/or liquid type. Another case comes from the glass container industry, in which costly changeovers are incurred in molding lines due to differences in the container mold and/or in the glass color among products. Similarly, in automated foundries time and cost expenditures in setups are dependent on the sequence of changes both in the alloy type and piece molds triggered at casting machines. The problem of production sequencing is also important in the textile industry on spinning facilities. The planned production sequence of yarn packages define the required setups to change the fiber blend and also provoke adjustments in yarn machines. More real world examples are present in chemicals, drugs and pharmaceuticals, pulp and paper, and animal nutrition, among other industries.

From a research perspective, the aforementioned problems belong to the field of lotsizing and scheduling problems (LS). LS models are usually expressed in the form of mixed integer programming (MIP) formulations. The advances observed in mathematical programming in recent years combined with the increase in computational power (hardware) and in the quality of general

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purpose mixed-integer programming commercial solvers (software) allowed sequence independent LS problems to be solved efficiently using exact methods for reasonable size instances. However, the development of tighter mathematical formulations is still mandatory to reduce the running times needed to solve LS instances with sequencing decisions, particularly when dealing with real world constraints and problem sizes. As a result, both the complexity and inherent applicability to real world problems caused an increased enthusiasm from the research community to tackle LS problems with sequencing decisions. This interest is shown in the reviews by Drexl and Kimms (1997), Zhu and Wilhelm (2006), Jans and Degraeve (2008) and Quadt and Kuhn (2008) and especially by the recent special issue Clark, Almada-Lobo, and Almeder (2011). Researchers have been incorporating additional scheduling decisions and features into LS models to improve their realism and potential applicability. However, none of the aforementioned reviews focuses on modeling techniques to integrate sequencing decisions in LS models and their impact on the solution quality achieved.

In this paper we first propose a framework to classify discrete time models for LS with sequencing decisions using two main sequencing dimensions: technique and time structure. Only the most relevant models in each class are reviewed to show their main features and to highlight the differences among them. Besides reviewing the models present in the literature we also introduce a new polynomial-sized model formulation to the problem which uses commodity flow based constraints to eliminate disconnected subtours and allows for multiple lots of the same product within each time period.

The performance of the models reviewed in the context of the framework and also of the new formulation is assessed by solving large size instances of the problem using a mixed-integer programming commercial solver. During the computational experiments we analyze the trade-offs present in these different modeling approaches. First, we study the correlation between the complexity introduced by allowing more general sequences (e.g. product repetition) and the solution quality obtained when a time limit is imposed. Second, we compare the use of an exponential number of constraints and variables against the use of compact model formulations. We focus on running times and upper bounds since our goal is to test the capability of providing solutions to instances of real-world size. In addition, many solution procedures for LS combine heuristics with exact methods, such as the progressive interval heuristics and the 'exchange' (fix-and-optimize) improvement heuristic, which rely on the solution of a series of sub-MIPs, also depend on the generation of good upper bounds. Hence, this assessment of the formulations can potentially contribute to the identification of the potentially most efficient MIP formulations to be used in these hybrid methods.

Our contributions are as follows. We present a new classification framework to classify modeling approaches to LS with sequencing decisions. The new framework is used to survey and classify the different modeling approaches present in the literature grouping models into classes. We also introduce a new commodity flow based formulation to integrate sequencing decisions in discrete time LS models. Finally, the extensive computational results present an evaluation of the pros and cons of the different modeling techniques, comparing models which, to the best of our knowledge, had never been compared. This enabled us to pinpoint the most efficient models in the several contexts studied.

The remainder of this paper is organized as follows. Section 2 presents the proposed classification framework for the modeling approaches. In Section 3 we describe the problem under study and all the assumptions made. Following the classes defined in our framework, Sections 4 and 5 present the reviewed models, as well as introduce the new formulation proposed herein. Computational experiments assessing the models' performance are shown in Section 6. Finally, Section 7 is devoted to final remarks, where conclusions from this work and some potential future research directions are highlighted.

2. Modeling sequence-dependent setups

In this section, we introduce a framework to classify the discrete time modeling approaches existing in the literature for LS with sequencing decisions. The framework is organized along two main sequencing dimensions: technique and time structure (see Fig. 1). A class is defined by the technique and time structure used, e.g. product oriented macro-period (PO–MP) models.

The sequence of production lots in a machine can be categorized following the definitions given by Kang, Malik, and Thomas (1999): a *production-sequence* refers to the sequence of products being produced on the machine over the entire planning horizon and a *period-sequence* denotes the sequence of setup states within a time period. In discrete time models for LS with sequencing decisions a production-sequence decomposes into period-sequences, hence the term sequence will be used hereafter to refer to period-sequences. The first dimension used for classification regards the technique used to capture sequencing decisions. Two main approaches are distinguished: product oriented (PO) and sequence oriented (SO) formulations. When using a PO technique, sequences are explicitly defined by the MIP model, while in SO formulations the MIP model prescribes for each period a sequence from a predetermined set of sequences, i.e. the model selects one sequence from the set.

Consider the representations of sequences depicted in Fig. 2. By definition a sequence is a connected direct graph where each node i represents a production lot of product i and arc (i,j) indicates a setup from product i to product j. Additionally, the dashed arcs identify the first (input arc) and the last (output arc) production lots in the sequence, i.e. the initial and final setup state of the machine. A SO formulation corresponds to the selection of a connected graph (sequence) to be applied in each time period, thus it does not require additional constraints to ensure the connectivity of the setup decisions. On the other hand, a PO formulation operates on the selection of arcs (setups) to be performed in each time period, hence the so-called disconnected subtour elimination constraints, which can be of an exponential size, are often required to ensure the connectivity of the subgraph induced by setup decisions. This is a major difference between these two approaches and explains why sequence oriented based formulations are easier to model. However, this potential advantage has the drawback of the number of possible sequences (decision variables) growing

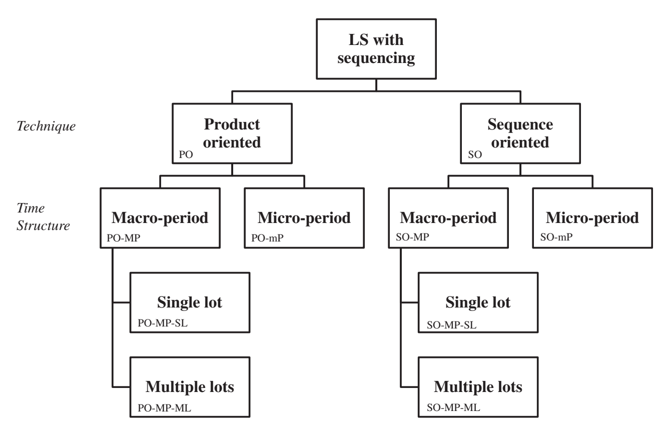

Fig. 1. Proposed classification framework.