European Journal of Operational Research 239 (2014) 711–730

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

# Robust ordinal regression for value functions handling interacting criteria

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ARTICLE INFO

*Article history:*
Received 23 February 2012
Accepted 18 May 2014
Available online 6 June 2014

*Keywords:*
Multiple criteria decision aiding
Value function
Interacting criteria
Robust ordinal regression

ABSTRACT

We present a new method called $UTA^{GMS}$–*INT* for ranking a finite set of alternatives evaluated on multiple criteria. It belongs to the family of Robust Ordinal Regression (ROR) methods which build a set of preference models compatible with preference information elicited by the Decision Maker (DM). The preference model used by $UTA^{GMS}$–*INT* is a general additive value function augmented by two types of components corresponding to "bonus" or "penalty" values for positively or negatively interacting pairs of criteria, respectively. When calculating value of a particular alternative, a bonus is added to the additive component of the value function if a given pair of criteria is in a positive synergy for performances of this alternative on the two criteria. Similarly, a penalty is subtracted from the additive component of the value function if a given pair of criteria is in a negative synergy for performances of the considered alternative on the two criteria. The preference information elicited by the DM is composed of pairwise comparisons of some reference alternatives, as well as of comparisons of some pairs of reference alternatives with respect to intensity of preference, either comprehensively or on a particular criterion. In $UTA^{GMS}$–*INT*, ROR starts with identification of pairs of interacting criteria for given preference information by solving a mixed-integer linear program. Once the interacting pairs are validated by the DM, ROR continues calculations with the whole set of compatible value functions handling the interacting criteria, to get necessary and possible preference relations in the considered set of alternatives. A single representative value function can be calculated to attribute specific scores to alternatives. It also gives values to bonuses and penalties. $UTA^{GMS}$–*INT* handles quite general interactions among criteria and provides an interesting alternative to the Choquet integral.



## 1. Introduction

Ranking, choice or sorting decision with respect to a finite set of alternatives evaluated on a finite set of criteria is a problem of uttermost importance in many real-world areas of decision-making (Ehrgott, Figueira, & Greco, 2010; Figueira, Greco, & Ehrgott, 2005). Among many approaches that have been designed to support the Multiple Criteria Decision Analysis (MCDA), three of them seem to prevail. The first one exploits the idea of assigning a score to each alternative, as it is the case of MAUT - Multi-Attribute Utility Theory (Keeney & Raiffa, 1976). The second relies on the principle of pairwise comparison of alternatives, as it is the case of outranking methods (Roy, 1996). The third one induces logical "if…, then…" decision rules from decision examples, as it is the case of DRSA – Dominance-based Rough Set Approach (Greco, Matarazzo, & Słowiński, 2001; Słowiński, Greco, & Matarazzo, 2009). The value function, the outranking relation and the set of decision rules are three *preference models* underlying these three main approaches. It is known that in order to build such models, the Decision Maker (DM) has to provide some preference information.

The *preference information* may be either direct or indirect, depending whether it specifies directly values of some parameters used in the preference model (e.g., trade-off weights, aspiration levels, discrimination thresholds, etc.) or whether it specifies some examples of holistic judgments from which compatible values of

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http://dx.doi.org/10.1016/j.ejor.2014.05.022
0377-2217/

the preference model parameters are induced. Eliciting direct preference information from the DM can be counterproductive in real-world decision-making because of a high cognitive effort required. Consequently, asking directly the DM to provide values for the parameters seems to make the DM uncomfortable. Eliciting indirect preference is less demanding of the cognitive effort. Indirect preference information is mainly used in the *ordinal regression* paradigm. According to this paradigm, a holistic preference information on a subset of some *reference* or *training alternatives* is known first and then a preference model compatible with the information is built and applied to the whole set of alternatives in order to arrive at a ranking, choice, or sorting recommendation.

Usually, from among many sets of parameters of a preference model representing the preference information given by the DM, only one specific set is selected and used to work out a recommendation. For example, while there exist many value functions representing the holistic preference information given by the DM, only one value function is typically used to recommend the best ranking, choice, or sorting of alternatives. Since the selection of one from among many sets of parameters compatible with the preference information given by the DM is rather arbitrary, *robust ordinal regression* proposes taking into account all the sets of parameters compatible with the preference information, in order to give a recommendation in terms of *necessary* and *possible* consequences of applying all the compatible instances of the preference model on the considered set of alternatives.

The recently proposed MCDA methods implementing robust ordinal regression on the three above-mentioned preference models have been described in Greco, Słowiński, Figueira, and Mousseau (2010). The first method in the series, called $UTA^{GMS}$ (Greco, Mousseau, & Słowiński, 2008), generalizes the *UTA* method (Jacquet-Lagrèze & Siskos, 1982) which applies ordinal regression to assess a set of additive value functions compatible with preference information provided by the DM. *UTA* aims at giving a complete ranking using one compatible value function, which is the one minimizing the sum of deviation errors or minimizing the number of ranking errors in the sense of Kendall or Spearman distance. In Jacquet-Lagrèze and Siskos (1982), the authors of *UTA* also recommend post-optimality analysis consisting in exploration of the vertices of the polyhedron of compatible value functions, in particular, the vertices for which one or more criteria get a maximum or minimum weight. $UTA^{GMS}$ is considering instead the whole set of compatible additive value functions to compute necessary and possible preference relations.

Even if the additive model is among the most popular ones, some critics have been addressed to this model because it has to obey an often unrealistic hypothesis about preferential independence among criteria. In consequence, it is not able to represent *interactions* among criteria. For example, consider evaluation of cars using such criteria as maximum speed, acceleration and price. In this case, there may exist a negative interaction (*negative synergy*) between maximum speed and acceleration because a car with a high maximum speed also has a good acceleration, so, even if each of these two criteria is very important for a DM who likes sport cars, their joint impact on reinforcement of preference of a more speedy and better accelerating car over a less speedy and worse accelerating car will be smaller than a simple addition of the two impacts corresponding to each of the two criteria considered separately in validation of this preference relation. In the same decision problem, there may exist a positive interaction (*positive synergy*) between maximum speed and price because a car with a high maximum speed usually also has a high price, and thus a car with a high maximum speed and relatively low price is very much appreciated. Thus, the comprehensive impact of these two criteria on the strength of preference of a more speedy and cheaper car over a less speedy and more expensive car is greater than the impact of the two criteria considered separately in validation of this preference relation.

To handle the interactions among criteria, one can consider *non-additive integrals*, such as Choquet integral and Sugeno integral (for a comprehensive survey on the use of non-additive integrals in MCDA see Grabisch, 1996). The non-additive integrals suffer, however, from some limitations within MCDA (Roy, 2009); in particular, they need that the evaluations on all criteria are expressed on the same scale. This means that in order to apply a non-additive integral it is necessary, for example, to estimate if the maximum speed of 200 km/h is as valuable as the price of 35,000€.

In this paper, we propose a new aggregation model which modifies the usual additive value function model so as to handle interactions among criteria without the necessity of expressing all the evaluations on the same scale. The paper is organized as follows. In the next section, we introduce main concepts and notation. In Section 3, first we show by an example what means violation of the preferential independence hypothesis for an additive value function, and then we continue the same example to show that the well known Choquet integral is not able to represent properly the observed interaction between criteria. In Section 4, we recall basic concepts and properties of robust ordinal regression. Our main proposal, extending $UTA^{GMS}$ to the case of interacting criteria, is given in Section 5. It starts with a reminder of the principle of robust ordinal regression, and then continues with presentation of the $UTA^{GMS}$–*INT* method. An illustrative example is provided in Section 6. The last section contains conclusions. All the proofs are deferred to the appendix.

## 2. Main concepts and notation

We are considering a multiple criteria decision problem where a finite set of alternatives $A = \{a, b, c, \ldots\}$ ($|A| = m$) is evaluated on a family of $n$ criteria $\{g_1, \ldots, g_i, \ldots, g_n\}$ ($I = \{1, \ldots n\}$). To simplify notation, we will identify the family of criteria with set $I$ of their indices. The family of criteria is supposed to satisfy consistency conditions (Roy & Bouyssou, 1993), i.e., completeness (all relevant criteria are considered), monotonicity (the better the evaluation of an alternative on considered criteria, the more it is preferable to another), and non-redundancy (no superfluous criteria are considered).

The set of all possible performances of alternatives on criterion $g_i$ is denoted by $X_i \subset \mathbb{R}$. Consequently, $X = \prod_{i=1}^{n} X_i$ is the performance space. We assume that $X_i = [x_{i*}, x_i^*]$, i.e., the performance scale of each criterion $g_i$ is bounded such that $x_{i*}$ and $x_i^*$ are the worst and the best performances, such that $x_* = (x_{1*}, \ldots, x_{n*})$ and $x^* = (x_1^*, \ldots, x_n^*)$ are two vectors of the best and the worst performances, respectively. We assume, without loss of generality, that the greater $g_i(a)$, the better alternative $a$ on criterion $g_i$; in other words, for any $a, b \in A$, any $i \in I$, performance $g_i(a) \in X_i$ is better than performance $g_i(b) \in X_i$ iff $g_i(a) > g_i(b)$. Each vector $x = (x_1, \ldots, x_n), x \in X$, is called performance vector, which may correspond to an alternative $a \in A$.

Multi-Attribute Utility Theory (MAUT), (Keeney & Raiffa, 1976) proposes to represent preferences of a Decision Maker (DM) on set $A$ of alternatives by an overall value function $U : X \rightarrow \mathbb{R}$, such that, for any pair of alternatives $a, b \in A : a \succsim b \iff U(a) \geqslant U(b)$, where $U(a)$ and $U(b)$ simplify the notation $U(g_1(a), \ldots, g_n(a))$ and $U(g_1(a), \ldots, g_n(a))$, respectively, and $\succsim$ is a weak preference relation on $A$, such that, for all $a, b \in A$, $a \succsim b$ means "$a$ is at least as good as $b$". As the role of value function $U$ is to aggregate vector performances of alternatives into a single real value, it is also called aggregation model or preference model.

One of the most popular value functions is the multiple attribute additive model: