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Decision Support

Analytic hierarchy process-hesitant group decision making

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ABSTRACT

In this paper, we consider that the judgments provided by the decision makers (DMs) cannot be aggregated and revised, then define them as hesitant judgments to describe the hesitancy experienced by the DMs in decision making. If there exist hesitant judgments in analytic hierarchy process-group decision making (AHP-GDM), then we call it AHP-hesitant group decision making (AHP-HGDM) as an extension of AHP-GDM. Based on hesitant multiplicative preference relations (HMPRs) to collect the hesitant judgments, we develop a hesitant multiplicative programming method (HMPM) as a new prioritization method to derive ratio-scale priorities from HMPRs. The HMPM is discussed in detail with examples to show its advantages and characteristics. The practicality and effectiveness of our methods are illustrated by an example of the water conservancy in China.

1. Introduction

Analytic hierarchy process (AHP) (Saaty, 1977, 1980a) is a popular and powerful technique for decision making, which is built on the human being's intrinsic ability to structure his/her perceptions or ideas hierarchically. Through pairwise comparisons of similar things against a given criterion, the decision makes (DMs) can provide judgments to represent the intensity of the importance of one thing over the other. AHP has wide applications in decision making problems (Dong, Hong, Xu, & Yu, 2013; Forman & Peniwati, 1998; Grošelj & Zadnik Stirn, 2012; Vaidya & Kumar, 2006; Xu, 2000; Xu & Wei, 1999).

To deal with the AHP problems, Saaty (1980a) proposed four basic steps. (1) Modeling: it involves the construction of a hierarchy at different levels of criteria, subcriteria and alternatives. (2) Valuation: based on a 1–9 ratio-scale measure, the DMs provide judgments over paired comparisons of objectives at each level of the hierarchy. (3) Prioritization: using prioritization methods to derive local priorities of the objectives at each level of the hierarchy. (4) Synthesis: using aggregation procedures (such as the weighted arithmetic average and the geometric mean) to synthesize the local priorities into global priorities of the alternatives.

In AHP-group decision making (AHP-GDM) (Ramanathan & Ganesh, 1994; Saaty, 1989), the DMs are forced to aggregate their individual judgments into group judgments to describe the relationship between the compared objectives. However, if the DMs cannot reach consensus with respect to the aggregated judgment(s), the confidence of the DMs to the final results may be reduced. This similar problem has also been considered by Hauser and Tadikamalla (1996). They claimed that many circumstances make such an aggregation process difficult. For example, with respect to an aggregated judgment, the DMs cannot agree with it in any case, but prefer to retain their original judgments.

In the existing works, the common solution to this problem is to revise the DMs' judgments to establish consensus paths, which is called a consensus reaching process. For example, Altuzarra, Moreno-Jiménez, and Salvador (2010) developed a Bayesian approach; Dong, Zhang, Hong, and Xu (2010) proposed some consensus models. If the DMs are not willing to revise their judgments, then the interval judgments that use intervals of numerical values can be applied to represent the margins of errors in judgments. For example, Saaty and Vargas (1987) originally proposed interval judgments in AHP, and computed the probability of rank reversal of the compared objectives.

However, if we consider AHP as another way to access an additive value function, it should be held to the standard as no rank-reversal. As early as 1982, Kamenetzky (1982)'s seminal work lays this out nicely. Moreover, Forman and Gass (2001) argued that the ratio-scale measure convey more information than the interval measure, and AHP must use ratio-scale priorities for the objectives above the lowest level of the hierarchy.

Following the traditional ratio-scale measure, and if the original judgments provided by the DMs cannot be aggregated and revised, is it possible to generate reasonable ratio-scale priorities of the compared objectives? In this paper, we define a hesitant judgment

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that includes several possible values to indicate the original judgments provided by the DMs, where the values in the hesitant judgment cannot be aggregated and revised. The hesitant judgments can be used to describe the hesitancy experienced by the DMs when they make comparisons. Then we develop a hesitant multiplicative programming method (HMPM) as a new prioritization method to derive ratio-scale priorities from hesitant judgments. Since all the original possible judgments provided by the DMs are preserved without aggregations and revisions, the overall confidence of the DMs to the results produced by the HMPM should not be reduced.

With respect to AHP-GDM, if there exist hesitant judgments, then we call it AHP-hesitant group decision making (AHP-HGDM) as an extension of AHP-GDM. To systematically introduce this topic, the remainder of this paper is organized as follows. Section 2 introduces some basic concepts related to hesitant judgments. Section 3 develops the HMPM. Section 4 gives some necessary discussions. In Section 5, we give a real-life example to illustrate our results. Section 6 summarizes this paper and offers some concluding remarks.

2. Hesitant multiplicative sets and hesitant multiplicative preference relations

Saaty (1980b) gave a 1–9 ratio scale (see Table 1) as a basis for the decision makers (DMs) to provide judgments over paired comparisons of objectives. Then a common tool, multiplicative preference relations (MPRs), is used to collect the judgments. With respect to a set of objectives $X = \{x_1, x_2, \ldots, x_n\}$, a multiplicative preference relation (MPR) can be denoted by $A = (a_{ij})_{n\times n}$ with the conditions that $a_{ij}a_{ji} = 1$, $a_{ij} \in [1/9, 9]$, where a_{ij} (a crisp judgment) indicates the degree that x_i is preferred to x_j. $a_{ij} = 1$ indicates that there is indifference between x_i and x_j; $a_{ij} > 1$ indicates that x_i is preferred to x_j; $a_{ij} < 1$ indicates that x_j is preferred to x_i.

If $A = (a_{ij})_{n\times n}$ is a consistent MPR, then it should satisfy the property as

$$a_{ij} = \frac{w_i}{w_j}, \quad i,j = 1,2,\ldots,n \tag{1}$$

where $w = (w_1, w_2, \ldots, w_n)$ is the priority vector of the objectives satisfying $\sum_{i=1}^{n} w_i = 1, w_i \geqslant 0$, $i = 1,2,\ldots,n$.

The idea of introducing hesitancy can refer to hesitant fuzzy sets (Torra, 2010), which is a hot topic recent years (Xu & Xia, 2011; Zhu & Xu, 2013, 2014; Zhu, Xu, & Xia, 2012, 2013). Since hesitant fuzzy sets have the advantage to handle imprecise whereby two or more sources of vagueness appear simultaneously, Xia and Xu (2013) extended this concept of hesitancy to MPRs, and developed the concepts of hesitant multiplicative sets (HMSs) and hesitant multiplicative preference relations (HMPRs). For our purpose, we now restate these concepts as follows.

Definition 1. Let X be a fixed set, a HMS is defined as

$$Z = \{< x,\ z(x) > | x \in X\} \tag{2}$$

where $z(x)$ is a subset of $[1/9, 9]$ following the 1–9 ratio scale.

Table 1
The 1–9 ratio scale.

Scale	Meaning
1	Equally preferred
3	Moderately preferred
5	Strongly preferred
7	Very strongly preferred
9	Extremely preferred
Other values between 1 and 9	Intermediate values used to represent compromise

For convenience, $z = z(x)$ can be called a hesitant multiplicative element (HME). Since a HME may consist of several possible values, it can be considered as a hesitant judgment in the decision making environment. Based on HMEs, hesitant multiplicative preference relations (HMPRs) can be defined as follows.

Definition 2. With respect to a set of objectives $X = \{x_1, x_2, \ldots, x_n\}$, a HMPR is defined as $Z = (z_{ij})_{n\times n}$, where $z_{ij} = \left\{ z_{ij}^{(l)} | l = 1, \ldots, |z_{ij}| \right\}$ is a HME indicating the preference degree(s) of x_i over x_j with the following conditions:

$$z_{ij}^{\rho(l)} z_{ji}^{\rho(l)} = 1, \quad z_{ii} = 1, \quad i,j = 1,2,\ldots,n; \tag{3}$$

$$z_{ij}^{\rho(l)} < z_{ij}^{\rho(l+1)},\ i < j \tag{4}$$

where $z_{ij}^{\rho(l)}$ is the ρth element in z_{ij}.

Specially, if there are missing elements in $Z = (z_{ij})_{n\times n}$, then it reduces to an incomplete HMPR; if $|z_{ij}| = 1$ for all $i,j = 1,2,\ldots,n$, then it reduces to a MPR.

3. Hesitant multiplicative programming method

The hesitant multiplicative programming method (HMPM) is a prioritization method, which arises from a fuzzy programming method (FPM) originally introduced by Mikhailov (2000). The FPM transforms the prioritization problem into a fuzzy programming problem that can easily be solved as standard linear programming. The FPM has some attractive properties, such as simplicity in computation, good precision and rank preservation. As an extension of the FPM, the HMPM preserves these advantages, and can reduce to the FPM in some cases.

To develop the HMPM, we first give its geometric representation in dealing with a prioritization problem. Considering some decision makers (DMs) involved in a decision making problem, they provide hesitant judgments over paired comparisons of objectives to construct a HMPR, $Z = (z_{ij})_{n\times n}$, where $z_{ij} = \left\{ z_{ij}^{(l)} | l = 1, \ldots, |z_{ij}| \right\}$. Motivated by Eq. (1), if there exists a priority vector $w = (w_1, w_2, \ldots, w_n)$, where $\sum_{i=1}^{n} w_i = 1$, $w_i \geqslant 0$, $i = 1,2,\ldots,n$, then we define Z as a consistent HMPR if

$$\frac{w_i}{w_j} = z_{ij}^{(1)}\ or \ldots or\ z_{ij}^{|z_{ij}|}, \quad i,j = 1,2,\ldots,n \tag{5}$$

which is called a consistency property of HMPRs.

According to Eq. (5), and let $R_{ij}(w) = w_i - w_j(z_{ij}^{(1)}\ or \ldots or\ z_{ij}^{|z_{ij}|})$, we define a hyperplane in a n-dimensional priority space as

$$G_{ij}(w) = \{w | R_{ij}(w) = 0\} \tag{6}$$

Let $G_o(w) = \{w^0 | w_1 + w_2 + \ldots + w_n = 1\}$ be the simplex hyperplane. Since the priority vector w must lie on $G_o(w)$, we further consider the intersection between $G_{ij}(w)$ and $G_o(w)$, which is defined as a hyperline:

$$L_{ij}(w) = \min(G_{ij}(w), G_o(w)) \tag{7}$$

Then the intersection of all hyperlines $L_{ij}(w)$ ($i,j = 1,2,\ldots,n$, $i < j$) is the solution of this prioritization problem, denoted by

$$p(w) = \min\{L_{ij}(w) | i,j = 1,2,\ldots,n, i < j\} \tag{8}$$

The geometric representation of the HMPM above is associated with the consistent case following Eq. (5). However, in nontrivial practical situations, most inconsistent cases approximately satisfy Eq. (5), which can be described as a fuzzy equality:

$$R_{ij}(w) \cong 0 \Longleftrightarrow w_i - w_j(z_{ij}^{(1)}\ or \ldots or\ z_{ij}^{|z_{ij}|}) \cong 0 \tag{9}$$