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Decision Support

Impulse control of pension fund contributions, in a regime switching economy

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ABSTRACT

In defined benefit pension plans, allowances are independent from the financial performance of the fund. And the sponsoring firm pays regularly contributions to limit deviations of fund assets from the mathematical reserve, necessary for covering the promised liabilities. This research paper proposes a method to optimize the timing and size of contributions, in a regime switching economy. The model takes into consideration important market frictions, like transactions costs, late payments and illiquidity. The problem is solved numerically using dynamic programming and impulse control techniques. Our approach is based on parallel grids, with trinomial links, discretizing the asset return in each economic regime.

1. Introduction

The world of pension provisions is currently shifting from unfunded social security towards private funding. In this context, the actuarial profession has a strong interest in the funding of pension plans and in timing of contributions payment. Pension funds are either classified as defined contribution or as defined benefit plans. They differ in risk and benefits. In defined contribution schemes, the financial risk is borne by affiliates and benefits directly depend upon assets performance. Whereas, in the other category of plans, this risk is borne by the sponsoring firm of the fund: allowances are warranted and independent from assets returns. For both classes of pension funds, contributions paid a long time before an employee's retirement, earn higher capital gains than most recent ones. But as they are immediate charges affecting the income statement of the sponsoring corporation, it is important to optimize the contribution schedule. This research paper studies this issue in presence of market frictions, for defined benefits pension funds.

Defined benefit pension plans have been extensively analyzed in the literature. Sundaresan and Zapatero (1997) argue that investors should maximize the expected utility of the surplus of assets over the liabilities of the fund. However, especially from the employer's point of view who pays for the defined benefit pension plan of his employees, the important issue is to find a contribution process which has small fluctuations and which leads as exactly as possible to the value of the mathematical reserve necessary for covering the liabilities promised in the pension plan. Therefore a whole branch of papers has studied the minimization of a loss function of contributions and the wealth to be obtained. In the papers of e.g. Haberman and Sung (1994, 2005), Boulier, Trussant, and Florens (1995), Josa-Fombellida and Rincôn-Zapatero (2004, 2006), the fund manager keeps the value of the assets as close as possible to liabilities by controlling the level of contributions. Cairns (1995, 2000) discusses the role of objectives in selecting an asset allocation strategy and has analyzed some current problems faced by defined benefit pension funds. Huang and Cairns (2006) or Hainaut and Deelstra (2011) study the optimal contribution rate for defined benefit pension plans when interest rates are stochastic.

But till now, this issue has mainly been studied with stable economic sources of randomness. The interested reader may e.g. refer to papers of Haberman and Sung (1994), Boulier et al. (1995), Cairns (2000) or Josa-Fombellida and Rincôn-Zapatero (2004, 2006, 2008, 2010) in which both contributions and assets allocation are optimized in continuous time and without transaction costs. In these works, the market is modeled by geometric Brownian motions. Even though this model is very popular, it is a well-known fact that pure diffusion processes are not an adequate representation of the characteristics of long term returns from risky assets. The papers of Ngwira and Gerrard (2007) or of Josa-Fombellida and Rincôn-Zapatero (2012) remedy to this drawback by adding jumps in assets returns and study the pension funding and asset allocation problem.

Jump-diffusion models represent a significant advance in research. But contrary to switching regime processes, they are

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partly unsatisfactory because they fail to duplicate economic cycles as stated by Henry (2009). Switching regime processes have already received a lot of attention in investment management practice with Hunt and Kavesh (1976), Hunt (1987) or Stovall (1996). Guidolin and Timmermann (2005) present evidence of persistent 'bull' and 'bear' regimes in UK stock and bond returns and considers their economic implications from the perspective of an investor's portfolio allocation. Similar results are found in Guidolin and Timmermann (2008), for international stock markets. Guidolin and Timmermann (2007) characterizes investors' asset allocation decisions under a regime switching model for asset returns with four states that are characterized as crash, slow growth, bull and recovery states. Cholette, Heinen, and Valdesogo (2009) fit skewed-t GARCH marginal distributions for international equity returns and a regime switching copula with two states. Al-Anaswah and Wilfing (2011) estimate a two-regimes Markov-switching specification of speculative bubbles. Hainaut and MacGilchrist (2012) study the strategic asset allocation between stocks and bonds when both marginal returns and copula are determined by a hidden Markov chain. On another hand, Calvet and Fisher (2001, 2004) shows that discretized versions of multifractal processes captures thick tails and have a switching regime structure. Finally, Hardy (2001) and the society of actuaries (SOA) since 2004, recommends switching processes to model long term stocks return, in actuarial applications. Frauendorfer, Jacoby, and Schwendener (2007) or Korn, Tak Kuen, and Zhang (2009) adopted this approach to optimize assets allocation in defined contribution pension plans.

Defined benefits pension plans are funded by contributions paid in by their sponsoring firm (and/or employees) and by the return on the invested capital. This work proposes a method to optimize the timing and size of these payments, whether fund assets are driven by a switching regime diffusion and in presence of market frictions. It contributes to the literature in several directions. First, research papers cited in this introduction optimizes payments in continuous time and without transaction costs. These unrealistic assumptions are removed in the studied framework. Instead, contributions are here controlled impulses, paid at discrete times, when assets deviate too much from liabilities. And transaction costs are both fixed and proportional to the volume of assets purchased or sold. The solving approach is based on dynamic programming and inspired from the works of Korn (1998, 1999) and Costabile, Leccadito, Massabo, and Russo (2014). The model takes also into consideration market imperfections. The first one is late payments, when delays are distributed as an exponential random variable. The second one is illiquidity that entails as underlined by Cont (2014), a relation between volume of assets purchased or sold and prices. In both cases, the impulse control strategy is adjusted to partly anticipate the impact of these frictions. Finally, this work proposes a method to calculate probabilities that the sponsor contributes to the fund over a certain time horizon. These probabilities are interesting management tools, not available when contribution calls are modeled by a continuous process.

The remainder of the paper is organized as follows. First, the features of the pension fund and the dynamics of its assets are introduced. Next, the Markov chain defining parameters in each economic regime is detailed. The third and fourth sections develop respectively the objective of the fund and the dynamic programming equation. These are followed by a paragraph detailing the numerical method based on parallel grids. Sections 7 and 8 respectively adapt the solving algorithm to take into account illiquidity risk and delay of payments. They are followed by a paragraph developing a method to estimate probabilities of impulse. The paper is concluded by a numerical illustration, in which the return of assets is modeled by a four states switching regime diffusion, fitted to CAC 40 daily returns.

2. The defined benefit pension fund

We consider a defined pension fund during the accumulation phase, that pays benefits at maturity T. The value of actuarial commitments, accounted as a liability in the balance sheet of the pension fund, is noted $R(t)$. This actuarial liability (also called technical provision, or mathematical reserve) $R(t)$ is the sum of expected discounted benefits, earned by employees at time t. We assume that no benefits is paid during the accumulation phase.

Assets managed by the pension fund are financed by a sponsor which is usually a private company outsourcing its pension liabilities. During the accumulation phase, to face the growth of charges related to retirement of employees, the sponsor regularly contributes to the fund. These capital injections or withdrawals limit deviations of fund assets from the mathematical reserve, necessary for covering the promised liabilities. But the timing and amounts paid in depend widely on the performance of assets. These assets are most of a time a basket of stocks and bonds, regularly rebalanced so as to maintain a constant proportion of stocks. The categories of assets and their percentages (the so called asset-mix) are defined in the mandate of management, that formally links the sponsor and the pension fund. As the definition of the asset mix sets indirectly by the same occasion the expected return and risk of assets, we focus in the remainder of this work on the optimization of the schedule and size of contributions. The influence of different sources of incompleteness are studied in the following sections. But we first only consider transaction costs.

In the remainder, A_t denotes the market value of assets managed by the pension fund. This is a stochastic process, defined on a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}\}_t, P)$. And its dynamics is ruled by an observable Markov chain, α_t defined on the same probability space. This chain carries on information about the current economic conjuncture and takes a finite number of values, noted N. Each value corresponds to a certain state of the economy (e.g. bull or bear market) and sets the average return and the volatility of assets. The features of α_t are developed in the next section. Before continuing, we define what we call an impulse strategy.

The assets are supplied by contributions at discrete times. An impulse strategy to contribute, $S = (\tau_n, \delta_n)$, consists in a sequence such that for all $n \in \mathbb{N}$, the times τ_i are stopping w.r.t. the filtration $(\mathcal{F}_t)_{t \geqslant 0}$. τ_n is the nth time at which the sponsor pays in a contribution to purchase new assets. And $\delta_n > 0$ defines the size of this contribution, that is measurable w.r.t. the sigma algebra of τ_n past $\mathcal{F}_{\tau_n}$ "control actions". The set of admissible impulse strategies is noted $\mathcal{A}$.

The market value of assets is driven by a switching diffusion process X_t^A defined as follows:

$$dX_t^A = \mu_A(\alpha_t)dt + \sigma_A(\alpha_t)dW_t^A \tag{2.1}$$

where $\mu_A(\alpha_t)$, $\sigma_A(\alpha_t)$ are function of the Markov chain α_t, representative of the economic situation. W_t^A is here a Brownian motion. The initial value of X_t^A is set to $\ln A_0$. The calibration of such type of processes to real time series is done with the Hamilton's filter (1989), reminded in Appendix A. An application using the filter is presented in Section 10. If no contribution is paid in till time t, the market value of assets is equal to:

$$A_t = e^{X_t^A} = e^{X_0^A + \int_0^t \mu_A(\alpha_s)ds + \int_0^t \sigma_A(\alpha_s)dW_s^A}. \tag{2.2}$$

If the sponsor supplies a net contribution I_t at time t (by net, we mean after transaction costs), the assets market value increases of:

$$A_t = A_{t-} + I_t. \tag{2.3}$$

But instead of working with absolute amount of money, we translate this contribution as a jump in the assets return, noted δ_t and calculated as follows: