European Journal of Operational Research 239 (2014) 830–841

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Spanning trees with variable degree bounds ☆

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ARTICLE INFO

Article history:
Received 27 December 2012
Accepted 17 May 2014
Available online 2 June 2014

Keywords:
OR in telecommunications networks
Spanning tree
Degree constraints
Wireless mesh networks

ABSTRACT

In this paper, we introduce and study a generalization of the degree constrained minimum spanning tree problem where we may install one of several available transmission systems (each with a different cost value) in each edge. The degree of the endnodes of each edge depends on the system installed on the edge. We also discuss a particular case that arises in the design of wireless mesh networks (in this variant the degree of the endnodes of each edge depend on the transmission system installed on it as well as on the length of the edge). We propose three classes of models using different sets of variables and compare from a theoretical perspective as well as from a computational point of view, the models and the corresponding linear programming relaxations. The computational results show that some of the proposed models are able to solve to optimality instances with 100 nodes and different scenarios.

1. Introduction

The Degree Constrained Minimum Spanning Tree Problem (DCMSTP) is a well known variant of the classical Minimum Spanning Tree problem. The DCMSTP contains additional constraints imposing a maximum value on the degree of the nodes (see for example Caccetta & Hill, 2001; Cunha & Lucena, 2007; Knowles, Corne, & Oates, 2000). Another variant imposing a minimum degree in all nodes except the leaves has been proposed in Almeida, Martins, and Souza (2012).

In this paper, we introduce and study a generalization of the DCMSTP where we may install one of several available transmission systems (each with a different cost value) in each edge. The degree of the endnodes of each edge depends on the system installed on the edge. We also discuss a particular case of the problem introduced here that arises in the design of wireless mesh networks. In this variant the degree of the endnodes of each edge depend on the transmission system installed on it as well as on the length of the edge (shorter edges and/or with a better transmission system allow higher degrees on its endpoints).

The paper is organized as follows. Section 2 describes the new problem and Section 2.1 describes the variant arising in the context of the design of wireless mesh networks. In Section 3, we describe three classes of models for the problems and compare them from a theoretical perspective. In Section 4 we present computational results for the general variant as well as for the wireless based variant to compare the models in terms of the Linear Programming gaps and running times to obtain the optimal solutions. Finally, Section 5 summarizes the main conclusions of this work.

2. Description and motivation of the problem

Consider an undirected graph $G = (X, E)$ where $X = \{1, \ldots, n\}$ represents the set of network nodes and $E \subseteq X^2$ is the set of edges $\{i, j\}$, representing possible network links (we denote by $E(i)$ the set of edges incident in node i). We assume that S is the set of available types of transmission systems that may be used in the network design solution. For each link $\{i, j\}$ and each transmission system $s \in S$, we associate a cost C_{ij}^s and a maximum degree D_{ij}^s of its endnodes i and j (typically, $D_{ij}^{s+1} > D_{ij}^s$ and $C_{ij}^{s+1} > C_{ij}^s$). Note that for a given transmission system s, the values D_{ij}^s and C_{ij}^s may differ for different pairs of nodes i and j. Also, it may happen that for some pairs, i and j, a given system s is not available and, in fact, E is the set of pairs of nodes such that at least one of the available transmission systems can be used.

We aim to find a "minimum" cost tree that satisfies the required degree constraints. Note that, for each edge such that more than one transmission system can be used, we have the option of installing a more expensive transmission system, allowing both endnodes to have higher degrees, or alternatively the

☆ This work is supported by National Funding from FCT – Fundação para a Ciência e a Tecnologia, under the project: PEst-OE/MAT/UI0152 and by the Vienna Science and Technology Fund (WWTF), under the project ICT10-027.
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http://dx.doi.org/10.1016/j.ejor.2014.05.034
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option of installing a lower cost transmission system constraining the degree of the endnodes to be lower.

Note that, when we only have one transmission system, say $s = s^*$, and $D_{ij}^{s^*}$ is the same for all links $\{i,j\}$, we obtain the DCMSTP mentioned in the previous section. Thus, the problem as described is *NP*-Hard (see Garey & Johnson, 1979).

Next, we explain that this problem is closely related to the design of point-to-point wireless networks when C_{ij}^{s} is constant for all $\{i,j\}$ such that the transmission system $s \in S$ can be used.

2.1. Wireless networks variant

In the network design of *point-to-point* wireless mesh networks, each link is implemented through a point-to-point wireless system composed by a pair of transmitter/receiver antennas and signal processing units (one at each endnode of the link) working on a frequency channel, chosen from a possible set of channels. A wireless system has always an associated distance range (*i.e.*, maximum distance between antennas), defined in its technical specification, which is roughly the same for all wireless channels and, in general, systems with higher distance ranges are more expensive. Therefore, given a set S of available wireless systems, the ones that can be installed on link $\{i,j\}$ are the ones whose distance range is not lower than the line of sight (*i.e.*, with no wireless obstacles between them) distance between i and j.

The distance range of a wireless system, though, always assumes no wireless interference from other sources. In current wireless technologies, due to the scarcity of the spectrum, there is a limited set of available frequency channels and many of them are partially overlapped between each other. For example, in IEEE 802.11 WiFi wireless mesh technologies, there is a total of 13 frequency channels, numbered from 1 to 13, but the maximum number of non-overlapped channels is three (for example, channels 1, 6 and 11) (IEEE 802.11 Working Group, 2003). If the network is configured with only non-overlapping channels, the maximum degree of the solution is quite constrained, which might not be a problem if graph G is dense but might be unfeasible when graph G is sparse.

In this paper, we address the variant with a possible overlapping set of frequency channels on each node. The usage of overlapping channels let the number of channels used by the links starting/ending on the same node to be higher (and, therefore, the node degree can be higher on a feasible solution) but the adjacent channel interference must be taken into consideration (see, for example, Liu, Venkatesan, & Cheng, 2009). A node with wireless links for different neighbor nodes uses different frequency channels. In a node using partially overlapped adjacent channels to different neighbor nodes, part of the transmitted signal on one channel is added as interference to the received signal on the other channels. The effect of this interference is that the distance range of the wireless systems is shortened. Note that if more wireless links are set on the edges adjacent to a node, the frequency channels must be closer between each other and more adjacent interference is added to each wireless system.

The maximum amount of interference of the other channels can be used to determine the resulting reduced distance range for each wireless system belonging to S for each possible node degree value (see Chen, Hugel, & Dressler (2011) and Villegas, López-Aguilera, Vidal, & Paradells (2007) for methods and models to estimate adjacent-channel interference). Then, for each pair of network nodes i and j, we can determine the wireless systems that can still be installed based on the distance between i and j and for each possible node degree value.

For example, a given system link s, with cost f_s and with a distance range of 15 when there is no interference, may have the distance range reduced to 5 if one (or both) of the endnodes of the link where we want to install the system has a degree of 3 or 4 or might not work at all if one of the endnodes has a degree larger than 4. In this case, if we want to install a system of type s on the link between a given pair of nodes i and j, whose distance is 10, for example, then we have $C_{ij}^{s} = f_s$ and $D_{ij}^{s} = 2$. On the other hand, if the distance between i and j is 4, for example, then we have $C_{ij}^{s} = f_s$ and $D_{ij}^{s} = 4$. Finally, the system s cannot be used if the distance between i and j is 20, for example.

Consider the example in Fig. 1 where the values associated with each link in Fig. 1a represent the link distances and the maximum degree (given by the number of overlapping frequencies that the operator may use) is 3. In this example, there are three available system types costing 5 (type I), 9 (type II) and 12 (type III). In links with a distance value less than 5, all system types can be used with the maximum degree in their endnodes. In links with a distance value between 5 and 10, systems of type I require a maximum degree of 2. Finally, in links with distance value between 10 and 15, systems of type I cannot be used (their distance range is lower) and systems of type II require a maximum degree of 2.

If we try to use a system of type I in the link $\{b,d\}$ there is no solution. If we use a system of type II in this link, the best solution has a cost of 48 (see the solution in Fig. 1b). If we upgrade the system installed on this link, using a system of type III, we obtain a solution with a lower cost of 47 (see solution in Fig. 1c) by replacing link $\{a,c\}$ (with a system of type II installed), by the link $\{a,b\}$ (with a system of type I installed). The upgrade of the system on the link $\{b,d\}$ allowed the degree of node b to be increased from 2 to 3.

In this problem, we assume that interference is critical only between channels used on the same node. In fact, a channel used on a wireless link between node i and j might also produce co-channel interference on other nodes whose wireless links (not involving i and j) are set on the same frequency channel. We consider, though, that such interference is negligible since, in general, the directionality of the antennas concentrates the wireless signal power in the direction towards the receiver antenna and attenuates strongly the signal towards other directions.

Note that the complete design of wireless mesh networks involves a larger set of issues like node location or channel assignment (see Bosio, Eisenblatter, Geerdes, Siomina, & Yuan, 2010 in applying mathematical optimization models in the design of WLANs). In this paper, we assume that node location is already decided. Although the variant addressed in this paper may be viewed as a simpler version of the problem since no channel assignment is performed, it still contains node degree constraints on the pair of network nodes that can be connected by a wireless system, that have not been considered before in the combinatorial optimization/network design area.

3. Formulations

In this section we describe three classes of integer linear formulations for the problem. The formulations described in this section contain a set of variables and inequalities that is common to all of them. These constraints are described in Section 3.1 and contain the binary variables $x_{\{i,j\}}$ indicating whether edge $\{i,j\} \in E$ is included in the solution and binary variables y_i^d indicating whether node $i \in X$ has degree equal to $d \in \{1,\ldots,D\}$ in the solution. These variables and similar constraints have already been used in the models introduced and described in Duhamel, Gouveia, Moura, and Souza (2012), Gouveia and Moura (2010) and Gouveia, Moura, and Sousa (2010) for problems with non-linear costs associated to the node degrees.

The first class of formulations (described in Section 3.2) uses additional binary variables $v_{\{i,j\}}^{m}$ indicating whether the edge $\{i,j\} \in E$ is selected and the maximum degree of nodes i and j is