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Discrete Optimization

A "reduce and solve" approach for the multiple-choice multidimensional knapsack problem

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ABSTRACT

The multiple-choice multidimensional knapsack problem (MMKP) is a well-known NP-hard combinatorial optimization problem with a number of important applications. In this paper, we present a "reduce and solve" heuristic approach which combines problem reduction techniques with an Integer Linear Programming (ILP) solver (CPLEX). The key ingredient of the proposed approach is a set of group fixing and variable fixing rules. These fixing rules rely mainly on information from the linear relaxation of the given problem and aim to generate reduced critical subproblem to be solved by the ILP solver. Additional strategies are used to explore the space of the reduced problems. Extensive experimental studies over two sets of 37 MMKP benchmark instances in the literature show that our approach competes favorably with the most recent state-of-the-art algorithms. In particular, for the set of 27 conventional benchmarks, the proposed approach finds an improved best lower bound for 11 instances and as a by-product improves all the previous best upper bounds. For the 10 additional instances with irregular structures, the method improves 7 best known results.

1. Introduction

The multiple-choice multidimensional knapsack problem (MMKP) can be informally described as follows. We are given a set of items that are divided into several groups and different types of limited resources. Each item requires a certain amount of each resource and generates a profit. The purpose of the MMKP is to select *exactly* one item from each group such that the total profit of the selected items is maximized while the consumption of each resource does not exceed the given limit (knapsack constraints).

Formally, given $G = \{G_1, G_2, \ldots, G_n\}$ the set of n disjoint groups (i.e., $G_i \cap G_j = \emptyset$ for each $i, j, 1 \leqslant i \neq j \leqslant n$). Let $I = \{1, 2, \ldots, n\}$ be the group index set, $g_i = |G_i|$ the number of items of group $G_i \in G$, m the number of resource types, b^k the capacity of resource k $(1 \leqslant k \leqslant m)$, $p_{ij} \geqslant 0$ the profit of the j^{th} item of G_i, w_{ij}^k the consumption for resource k of the j^{th} item of G_i. Additionally, let x_{ij} be the decision variable such that $x_{ij} = 1$ if the j^{th} item of group G_i is selected; $x_{ij} = 0$ otherwise. Then the MMKP can be stated as follows:

$$max \sum_{i \in I} \sum_{j \in \{1,\ldots,g_i\}} p_{ij} x_{ij} \quad (1)$$

subject to:

$$\sum\nolimits_{i \in I} \sum_{j \in \{1\ldots g_i\}} w_{ij}^k x_{ij} \leqslant b^k, k \in \{1, \ldots, m\} \quad (2)$$

$$\sum\nolimits_{j \in \{1\ldots g_i\}} x_{ij} = 1, i \in I \quad (3)$$

$$x_{ij} \in \{0, 1\}, i \in I, j \in \{1, \ldots, g_i\} \quad (4)$$

The MMKP is tightly related to the conventional multidimensional knapsack problem (MKP) (Boussier, Vasquez, Vimont, Hanafi, & Michelon, 2010; Kellerer, Pferschy, & Pisinger, 2004; Puchinger, Raidl, & Pferschy, 2010; Vasquez & Hao, 2001; Vasquez & Vimont, 2005; Wilbaut & Hanafi, 2009) since the MMKP can be reduced to the MKP by restricting each group to a single item and dropping constraint (3). Like the MKP, the MMKP is known to be NP-hard. In addition to its theoretical importance, the MMKP is notable for its capacity of modeling a number of practical applications such as logistics (Basnet & Wilson, 2005), resource allocation (Gavish & Pirkul, 1982), capital budgeting (Pisinger, 2001), and telecommunications (Watson, 2001).

Compared with the conventional MKP, the MMKP is somewhat less studied until recently. Yet, given both its theoretical and practical relevance, the MMKP is receiving increasing attention in recent years and a number of effective solution approaches have been proposed in the literature. For instance, exact methods based on the branch and bound framework were reported in Ghasemi

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and Razzazi (2011), Khan (1998) and Sbihi (2007). These algorithms have the advantage of guaranteeing the optimality of the solution found. Unfortunately, due to the very high computational complexity of the MMKP, exact approaches apply only to instances of limited sizes (i.e., $n = 100$ and $m = 10$ for the instances we used). To handle larger instances, several heuristic approaches were developed to seek sub-optimal solutions (corresponding to lower bounds) in an acceptable computing time.

For instance, Hifi, Michrafy, and Sbihi (2006) introduced a reactive local search algorithm and showed much better results than those reported in Moser, Jokanovic, and Shiratori (1997), which was the first paper dealing directly with the MMKP. Later, the authors of Cherfi and Hifi (2009) proposed an efficient hybrid heuristic combining local branching with column generation techniques, which improved the lower bounds for many benchmark instances. In Hanafi, Mansi, and Wilbaut (2009) an iterative relaxation-based heuristic was applied to solve the MMKP, where a series of small sub-problems are generated by exploiting information obtained from a series of relaxations. In Crévits, Hanafi, Mansi, and Wilbaut (2012), the authors employed a similar but more general approach called semi-continuous relaxation heuristic approach where variables are forced to take values close to 0 or 1. More recently, again based on the iterative relaxation-based heuristic framework, the authors of Mansi, Alves, Valerio de Carvalho, and Hanafi (2013) explored a new strategy, consisting of a family of new cuts and a reformulation procedure used at each iteration to improve the performance of the heuristic and to define the reduced problem. This method reported most of the current best known results over the set of conventional MMKP benchmark instances, which will be used as one of our references for performance assessment and comparison. In Shojaei, Basten, Geilen, and Davoodi (2013), the authors proposed an original parameterized compositional pareto-algebraic heuristic (CPH) which explores incremental problem solving and parallelization. They reported interesting results on the well-known MMKP benchmark instances and introduced a set of new instances that we will use in our work. Finally, there are several other recent and interesting studies based on general approaches like ant colony optimization combined with local search (Iqbal, Faizul Bari, & Sohel Rahman, 2010), strategic oscillation exploring surrogate constraint information (Htiouech & Bouamama, 2013), Lagrangian neighborhood search (Hifi & Wu, 2012) and tabu search (Hiremath & Hill, 2013).

In this paper, we present a "reduce and solve" heuristic approach that jointly makes use of problem reduction techniques and the state-of-the-art CPLEX ILP solver. The basic idea of the proposed approach is to employ some dedicated heuristics to fix a number of groups and variables in order to obtain a reduced critical subproblem which is then solved by the ILP solver. The key issue is how to choose the groups and variables to fix. For this purpose, we first define general fixing rules based on information from linear relaxation. To better explore the space of the reduced problems and achieve improved results (lower bounds), we additionally introduce specific strategies to enlarge progressively the reduced subproblems which are to be solved by CPLEX. Notice that our group and variable fixing techniques are in connection with the notion of strongly determined and consistent variables (Glover, 1977, 2005). Similar strategies for temporary or definitive variable fixing are explored in other contexts like, for instance, 0–1 mixed integer programming and binary quadratic programming (Wang, Lu, Glover, & Hao, 2011, 2013; Wilbaut & Hanafi, 2009).

To assess the merit and limit of the proposed approach, we carry out extensive computational experiments based on two sets of benchmark instances from the literature. These experiments show that the proposed approach competes favorably with the state-of-the-art methods and is able to discover 11 improved lower bounds and in passing to improve all the current best upper bounds reported in the literature for the set of 27 conventional benchmark instances. Moreover the proposed approach improves 7 best known results for the 10 additional benchmarks with irregular structures.

The paper is organized as follows. In Section 2, we present in detail the proposed approach. We begin with the introduction of the group and variable fixing rules and then introduce two solution procedures for the exploration of different reduced problems. Section 3 is dedicated to an extensive computational assessment in comparison with the state-of-the-art approaches. We also show an analysis of the effect of the group and variable fixing techniques in Sections 4. Conclusions are given in the last section.

2. A "reduce and solve" approach for the MMKP

2.1. General approach

The "reduce and solve" approach proposed in this paper can be summarized as a three-step method.

1. Group fixing: This step aims to identify some variables which are highly likely to be part of the optimal solution and fixes them to the value of 1. Given the constraint (3), once a group has a variable assigned the value of 1, the remaining variables of the group must be assigned the value of 0. We remove then the group (the group is said fixed) from the initial problem P, leading to a first reduced problem P'. Let q be the number of fixed groups.
2. Variable fixing: For each of the $n - q$ remaining groups of the problem P', we identify some variables that are unlikely to be part of the optimal solution, fix these variables to 0 and remove them from problem P', leading to a further reduced problem P''.
3. ILP solving: We run CLPEX to solve P''.

Given this general procedure, it is clear that the success of this approach depends on the methods used for group fixing (step 1) and variable fixing (step 2). We will explain in Sections 2.3 and 2.4 the heuristic fixing rules based on linear relaxation of the problem. However, whatever the method we use, it is possible that some variables are fixed to a wrong value. To mitigate this risk, we introduce additional strategies to decrease gradually the number of fixed variables. By doing so, we explore different and increasingly larger reduced problems which provides a means to achieve improved solutions. These strategies are presented in Sections 2.5.2 and 2.5.3.

2.2. Basic definitions and notations

The following notations and definitions will be used in the presentation of the proposed approach.

- Let $G = \{G_1, G_2, \dots, G_n\}$ be the given MMKP problem P with its index set $I = \{1, 2, \dots, n\}$ and let x^* be an optimal solution of problem P.
- $LP(P)$ and $\bar{x}$ denote respectively the linear relaxation of P and an optimal solution of $LP(P)$.
- $\underline{v}(P)$ and $\bar{v}(P)$ denote respectively a lower bound and an upper bound of P.
- $(P \mid c), LP(P \mid c)$ and $\bar{x}^c$ denotes respectively the problem P with exactly one additional constraint c, the linear relaxation of $(P \mid c)$ and an optimal solution of $LP(P \mid c)$.
- Integer group: Given the LP-relaxation optimal solution $\bar{x}$ of $LP(P)$, a group G_i of G is called *integer group* in $\bar{x}$ if $\exists j_i \in \{1, 2, \dots, g_i\} : \bar{x}_{ij_i} = 1$.