European Journal of Operational Research 239 (2014) 335–348

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

MIP models for resource-constrained project scheduling with flexible resource profiles

Anulark Naber *, Rainer Kolisch

TUM School of Management, Technische Universität München, Arcissstr. 21, 80333 Munich, Germany

ARTICLE INFO

Article history:
Received 3 August 2013
Accepted 21 May 2014
Available online 3 June 2014

Keywords:
Project scheduling
Flexible profiles
Principal resources
Dependent resources
Discrete-time models

ABSTRACT

This paper addresses the resource-constrained project scheduling problem with flexible resource profiles (FRCPSP). Such a problem often arises in many real-world applications, in which the resource usage of an activity is not merely constant, but can be adjusted from period to period. The FRCPSP is, therefore, to simultaneously determine the start time, the resource profile, and the duration of each activity in order to minimize the makespan, subject to precedence relationships, limited availability of multiple resources, and restrictions on resource profiles. We propose four discrete-time model formulations and compare their model efficiency in terms of solution quality and computational times. Both preprocessing and priority-based heuristic methods are also applied to compute both upper and lower bounds of the makespan. Our comparative results show significant dominance of one of the models, the so-called "variable-intensity-based" model, in both solution quality and runtimes.

1. Introduction

The resource-constrained project scheduling problem (RCPSP) and its variants have been broadly investigated during the last several decades, since the first discrete-time model was introduced by Pritsker, Watters, and Wolfe (1969). The traditional RCPSP or PS/prec/C_{max} (Brucker, Drexl, Möhring, Neumann, & Pesch, 1999) deals with a set of *n* activities which are to be scheduled, such that the project completion time or makespan is minimized, subject to two main constraints: (1) the technological precedence constraints in which an activity cannot be started, until all of its preceding activities have been completed; and (2) the limited availability of resources, given only one mode for each activity. A mode is usually predefined as a nonpreemptive, constant resource usage of an activity over its entire predetermined fixed duration. When several modes and durations for each activity are considered, the problem becomes the multi-mode RCPSP (MRCPSP), a generalization of the RCPSP that additionally selects a mode for each activity. Several surveys on the single- and multi-mode RCPSPs have been conducted, most recently by Hartmann and Briskorn (2010) and Węglarz, Józefowska, Mika, and Waligóra (2011).

While in the RCPSP and MRCPSP the resource allocation over the duration of each activity is given and normally constant, Kolisch, Meyer, Mohr, Schwindt, and Urmann (2003) propose a model in which the resource allocation must be determined. As a result, the "work profile" defined by Kolisch et al. (2003) is no longer limited to a rectangular shape in the traditional sense. Related to the work profile, the "*work content*" (Fündeling & Trautmann, 2010) is defined as the total amount of resource required to complete an activity. For example, a work content of 10 man-days for an activity may be allocated into a constant profile of 2 men for 5 days as in the RCPSP, or a flexible profile of 3 men for 2 days and 2 men for 2 days. Since resources are not restricted only to human resources, we prefer to use more general terms, namely "*resource profile*" and "*resource requirement*", instead of work profile and work content, respectively.

By allowing resource allocation to take flexible forms, the new problem becomes a generalization of the RCPSP. Hence, its optimal makespan is at least as good as the makespan of RCPSP. Under these new circumstances, the resource usage in each time period and the duration of each activity are unknown a priori and, thus, need to be simultaneously determined while scheduling activities by their starting times. This problem is termed here as the RCPSP with **F**lexible resource profiles (FRCPSP) that is capable of handling dependency and independency of multiple resources and free-forms of profiles including uniform in the traditional sense.

The latest advanced computer technology has enabled and empowered many commercial optimization packages in solving large-scale, difficult mathematical models much more efficiently and reliably than ever before. Taking advantage of the developed

---

* Corresponding author. Tel.: +49 (89) 289 25161; fax: +49 (89) 289 25166.
E-mail addresses: anulark.naber@tum.de (A. Naber), rainer.kolisch@tum.de (R. Kolisch).

http://dx.doi.org/10.1016/j.ejor.2014.05.036
0377-2217/

tools, new models are formulated for the RCPSP and extensively compared by Koné, Artigues, Lopez, and Mongeau (2011) and Bianco and Caramia (2013). Both works substantiate that formulated models based on choices of decision variables and constraints do have significant effects on the efficiency of exact methods. Their computational results show that, for test instances with short processing times, the discrete-time models, in general, provide stronger LP relaxation bounds and shorter solution times, despite their high pseudo-polynomial number of binary variables.

Motivated by their works and limited existing research on the FRCPSP despite its tremendous potentials and applications, we propose, in this paper, four discrete-time model formulations for the FRCPSP and investigate the model efficiency in terms of problem size, solution quality, and runtime. To the best of our knowledge, such a complete study on the FRCPSP has not yet been published, and, thus, does contribute significantly in broadening the research scope in this area. It is expected that a strong model of the FRCPSP would also contribute to other related problems which the FRCPSP structure is either embedded in or considered as a relaxation problem of.

This paper is organized as follows. Section 2 describes the problem and assumptions. Section 3 briefly provides a literature review, whereas Section 4, the main section, proposes four model formulations. Section 5 describes our preprocessing calculations and priority-rule heuristic algorithm to compute the start and finish times of activities as well as the lower and upper bounds of makespan. Section 6 reports our computational comparisons and analyses. Concluding remarks are finally given in Section 7.

2. Problem description

The FRCPSP simultaneously determines an optimal schedule of nonpreemptive activities and their resource profiles that minimize the project makespan. Similar to the RCPSP, each activity requires, once started, one nonpreemptive profile per resource over its entire processing time. The project schedule must also observe all precedence requirements depicted in a precedence network, denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, in which $\mathcal{V}$ is the set of all activities which may include dummy source activity 0 and dummy sink activity $(n+1)$, where necessary, and $\mathcal{E}$ is the set of arcs representing standard precedence relationships, namely finish-start with zero time lag between two activities.

In addition, flexible profiles must satisfy the following practical constraints. (1) The total amount of each required resource assigned to each activity over its duration must at least satisfy its resource requirement. Implicitly, we assume that the duration of an activity results from a nonincreasing function of the amount of resource usage per time period. The more the allocated resource, the shorter the duration. (2) There must be at least a number of consecutive periods having a constant resource usage. This constraint is called the minimum block length (Fündeling & Trautmann, 2010). (3) The resource usage in a time period must be within a specified range designated by a lower and an upper bound. Without loss of generality, these bounds of a nondummy activity are assumed positive.

Unlike most of other similar research, all resource amounts are assumed here continuous and renewable with time-varying capacity (Klein, 2000). Obvious examples of continuous resources include electric, hydraulic or pneumatic power sources, fuel, computer memory, and CPU processing power. For employees or machines, which are normally assigned as discrete resources, may also be considered as continuous resources, when each employee or machine can process multiple activities in parallel. In such a case, the employee or machine is allotted in the (fractional) units of resource-time, such as man-days or machine-hours. In a period (day) of 8 hours, an employee may equally work on two activities in parallel, spending his/her time of 0.5 man-day to each activity. Additionally, the discrete nature of resources may be compromised, provided that the fractional part is trivial, when relatively compared with the high magnitude of resource quantity allotted, or when the resource planning is done at the tactical level, in which the required resources are just roughly estimated.

Note that traditional, constant profiles or modes as in the RCPSP and MRCPSP may also be enforced in the FRCPSP as a special case using equal lower and upper bounds of resource usage. At the outset, one may attempt to exhaustively generate resource profiles (modes) for each activity and convert the FRCPSP into the MRCPSP with flexible modes. However, the profile generation, a combinatorial problem in its own right, is only possible for discrete resources. For continuous resources addressed here, it deems impossible to facilitate such a problem conversion.

In this paper, one discrete-time system is commonly used for all activities and resources. As such, each activity must start at the beginning of a time period and finish at the end of a time period. Dummy activities, if any, are assumed to have zero resource requirements. The dummy source activity must start and end at the end of period 0, while the dummy sink starts and ends in the beginning of period (makespan + 1).

Resources required by an activity are classified here into three general categories, namely *principal*, *dependent*, and *independent* resources. To the best of our knowledge, such terminology has not yet been explicitly defined in the literature. Neither has the independent resource been explicitly addressed in the FRCPSP.

(1) A *principal* resource of an activity is the main resource whose usage amount may be depended upon or used as a computational basis by other resources to process the activity. Multiple principal resources may exist in a project, but only one principal is designated per activity. Fündeling and Trautmann (2010) called this type the work content resource.

(2) A *dependent* resource of an activity is a resource whose usage quantity depends on that of its principal resource to process the activity. Based on a fundamental assumption that the resource amounts are additive, the quantity of dependent resource *r* used by activity *j* in period *t*, denoted by q_{rjt}, follows a nondecreasing linear resource function of the quantity of its principal resource *k* used by activity *j* in period *t*, q_{kjt}. Fig. 1 illustrates an example of a linear resource function and corresponding profiles of both principal and dependent resources that satisfy such an assumption. Given that α_{krj} and β_{krj} denote the coefficient and the constant of a linear function, respectively, the quantity of resource *r* dependent on that of resource *k* is, therefore, $q_{rjt} = \alpha_{krj}q_{kjt} + \beta_{krj}$. To guarantee problem feasibility, a resource may be assigned to an activity more than its requirement, but all required resources must be allocated concurrently and nonpreemptively, once the activity starts and throughout the duration till its completion.

(3) An *independent* resource of an activity is a resource whose amount is independent from the quantity of any resources, although its timing must synchronize with the other resources required by the same activity.

As an example, one man-day of bio-lab technician as a principle resource $r = 1$ needs one day of fluorescence microscope as a dependent resource $r = 2$ during his/her activity *j* with the resource function $q_{2jt} = q_{1jt}$. If one man-day of the technician is allocated 30% and 70% to activities 1 and 2 that are being processed in parallel, respectively, the microscope is also time-shared proportionally to both activities. Similarly, a laboratory room, as a dependent