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Sensitivity analysis of the newsvendor model

Avijit Khanra *, Chetan Soman, Tathagata Bandyopadhyay

Indian Institute of Management, Ahmedabad 380015, India

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ABSTRACT

Quality of decisions in inventory management models depends on the accuracy of parameter estimates used for decision making. In many situations, error in decision making is unavoidable. In such cases, sensitivity analysis is necessary for better implementation of the model. Though the newsvendor model is one of the most researched inventory models, little is known about its robustness. In this paper, we perform sensitivity analysis of the classical newsvendor model. Conditions for symmetry/skewness of cost deviation (i.e., deviation of expected demand–supply mismatch cost from its minimum) have been identified. These conditions are closely linked with symmetry/skewness of the demand density function. A lower bound of cost deviation is established for symmetric unimodal demand distributions. Based on demonstrations of the lower bound, we found the newsvendor model to be sensitive to sub-optimal ordering decisions, more sensitive than the economic order quantity model. Order quantity deviation (i.e., deviation of order quantity from its optimum) is explored briefly. We found the magnitude of order quantity deviation to be comparable with that of parameter estimation error. Mean demand is identified as the most influential parameter in deciding order quantity deviation.

1. Introduction

The newsvendor problem is one of the most well-researched and widely applicable inventory management problems (Choi, 2012). It is about deciding the order quantity of a product with uncertain demand such that the penalty associated with demand–supply mismatch at the end of the selling season is minimized. This inventory problem was first addressed by Arrow, Harris, and Marschak (1951) in their seminal paper "Optimal Inventory Policy". Due to its wide applicability, different versions of the newsvendor model have been developed in the past six decades. Review of those works can be found in Silver, Pyke, and Peterson (1998), Khouja (1999), Qin, Wang, Vakharia, Chen, and Seref (2011), and Choi (2012). The simplest version of the newsvendor model, which is referred to as the classical newsvendor model (seeSilver et al., 1998, chap. 10 for assumptions and analysis), is focus of the current study.

Newsvendor model, like any other inventory optimization model, requires knowledge of certain parameters for decision making. One common implementation issue with these models is the possibility of error in estimation of the model parameters. The optimal order quantity, Q^* in the newsvendor model is given by $F(Q^*) = c_u/(c_o + c_u)$ where F denotes the demand distribution function and c_o and c_u denote unit over-stocking and under-stocking costs. Each of these parameters may not be known correctly in every situation. For example, newsvendor deals with stochastic demand, whose realization takes place only after the procurement decision is made. Hence, theoretically speaking, correct knowledge of the demand distribution function (i.e., the form and associated parameters) with certainty is impossible at the time of decision making.

Unit over-stocking cost is the unit purchase cost less salvage value of an unsold item. Though purchase cost is generally known at the time of procurement decision, there are exceptions. If the newsvendor itself is the manufacturer and the production environment is complex (involving multiple items), purchase cost, i.e., production cost specific to the focal item may not be known correctly. The other component of over-stocking cost, i.e., salvage value, too, may not be correctly known. Newsvendor sells excess inventory in secondary market and then the remaining stock is disposed off. This process begins after the selling season is over and it involves multiple agents, thereby making the assessment of salvage value at the time of procurement decision difficult.

Unit under-stocking cost is the sum of unit profit and goodwill loss due to unmet demand of one unit. Unit profit is the unit selling price less unit purchase cost. We have already noted that purchase cost may not be known correctly in some situations. If the selling price is market-driven (e.g., selling prices of commodity products), due to time-precedence of the procurement decision over selling

* Corresponding author. Tel.: +91 7405696960.
E-mail address: avijitk@iimahd.ernet.in (A. Khanra).

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season, actual selling price may not be correctly known at the time of decision making. The other component of under-stocking cost, i.e., stock-out goodwill loss is the most elusive cost component in the newsvendor model. Stock-out is reflected back as loss of future demand (of the concerned product as well as other products) through complex human behavior (Hadley & Whitin, 1963, pg. 20), which makes the quantification of goodwill loss very difficult.

It is evident from the above discussion that parameter estimation error is unavoidable in the newsvendor model. Then the operational decision, which is derived using the parameter estimates, is likely to deviate from the optimum. Parameter estimation error is not the only reason for sub-optimal decisions in the newsvendor model. Based on experiments, Schweitzer and Cachon (2000) reported that managers tend to order more than the optimum when $Q^* < \mu$ and order less than the optimum when $Q^* > \mu$ where μ denotes the mean demand. This "pull-to-center" effect was confirmed by other scholars (e.g., Bostian, Holt, & Smith, 2008). A number of alternate explanations were proposed, for example, Su (2008) showed that bounded rationality on the part of the newsvendor leads to the "pull-to-center" effect.

A sub-optimal decision, irrespective of the reason behind it, leads to greater cost. We study the nature of deviation of expected demand–supply mismatch cost from its minimum, in short, cost deviation. In particular, we attempt to address two issues. First, we identify the conditions for symmetry and skewness of cost deviation. If the newsvendor is uncertain about the optimal order quantity, these conditions would tell which is better, ordering more or ordering less. Second, we study the magnitude of cost deviation by establishing a lower bound. This reveals whether the newsvendor model is sensitive to sub-optimal decisions or not. Depending upon severity of the situation, further actions can be taken to improve the decision making.

The remainder of this paper is organized as follows. We review relevant literature and derive an expression for cost deviation before proceeding to the next section. In Section 2, we identify the conditions for symmetry/skewness of cost deviation. In Section 3, we establish a lower bound of cost deviation for symmetric unimodal demand distributions. Then we demonstrate the lower bound along with cost deviation for normal demand distribution. In Section 4, we briefly study order quantity deviation, i.e., deviation of order quantity from its optimum. We conclude in Section 5 by discussing main results of this work and identifying future research possibilities.

1.1. Related literature

Sensitivity analysis of some important inventory models are available in the literature. Economic order quantity (EOQ) model is one of the most popular inventory models. One reason for its popularity is its robustness to sub-optimal decisions. A discussion on sensitivity analysis of the EOQ model can be found in operations management textbooks (e.g., Nahmias, 2001). In the EOQ model, cost deviation is left-skewed and relatively insensitive to order quantity deviation. Zheng (1992) performed sensitivity analysis of the stochastic (r,Q) inventory system and found it to be more robust than its deterministic counterpart, the EOQ model. Chen and Zheng (1997) found similar result for the stochastic (s,S) inventory system with exponential demand.

A surprise omission from the above inventory models is the newsvendor model. However, there are some studies that indirectly address sensitivity analysis of the newsvendor model. We can classify them into two groups: (a) sensitivity of cost deviation to sub-optimal order quantities and (b) sensitivity of order quantity deviation to parameter estimation error. These two can be combined to yield sensitivity of cost deviation to parameter estimation error. We can further classify studies in each of the above groups into four subgroups: (i) direction of deviation, (ii) symmetry/skewness of deviation, (iii) magnitude of deviation, and (iv) distribution of deviation. Since the newsvendor model is multi-parameter model, the question of parameter importance (in influencing order quantity deviation or cost deviation) is relevant too.

Direction of cost and order quantity deviations can be understood easily by examining the standard results of the newsvendor model. Lapin (1988, chap. 6) noted down some of these. Since expected mismatch cost is convex in order quantity, it increases whenever order quantity deviates from its optimum. Note that cost deviation is positive if expected mismatch cost increases and negative if it decreases. The same is true for order quantity deviation. Since $F(Q^*) = c_u/(c_o + c_u)$, order quantity increases with underestimation of over-stocking cost (c_o) and decreases with its overestimation. This relation is exactly opposite for under-stocking cost (c_u). Order quantity is increasing in mean demand. Using mean preserving transformation, Gerchak and Mossman (1992) showed that order quantity is increasing in demand variability if $F(\mu) < \xi$ where $\xi = c_u/(c_o + c_u)$ is known as the critical fractile. The relation is opposite if $F(\mu) > \xi$.

Sensitivity analysis results regarding directions of deviations of some generalized newsvendor models are available in literature. The classical newsvendor model is a special case of the generalized models. Lau and Lau (2002) considered a supply chain with newsvendor type retailer and identified impact of demand variability on decision variables. Their conclusions are similar to that of Gerchak and Mossman (1992) when order quantity is the only decision variable, i.e., the retailer faces the classical newsvendor problem. Eeckhoudt, Gollier, and Schlesinger (1995) studied risk-averse newsvendor problem (risk-neutrality is equivalent to the classical problem) and identified influence of cost parameters on order quantity. Wang (2011) performed a similar study for newsvendor problem with advertisement-sensitive demand (no advertisement is equivalent to the classical problem). Their findings are similar to that of Lapin (1988).

In the newsvendor literature, there are studies that develop extensions of the classical newsvendor model and perform sensitivity analysis w.r.t. parameter(s) that differentiate it from the classical model, for example, Chen and Chen (2010) studied a multi-product budget-constrained newsvendor problem with a reservation policy and performed sensitivity analysis of expected profit and order quantity to the budget amount. There are more such studies. These studies, being contextually different, do not add much to our understanding of robustness of the classical newsvendor model.

Based on the above literature review, it is clear that our understanding of sensitivity of the classical newsvendor model is limited to the directions of deviations only. In this work, we study symmetry and magnitude of cost deviation as order quantity deviates from its optimum. We briefly explore order quantity deviation too.

1.2. Expression for cost deviation

We use the following notations in this paper.

a	Lower limit of demand. $a \geqslant 0$.
b	Upper limit of demand. $a < b < \infty$.
r	Ratio of demand limits. $r = a/b, r \in [0, 1)$.
$F()$	Distribution function of stochastic demand. $F(a) = 0, F(b) = 1$. We assume strict monotony of F in $[a, b]$ so that the optimal solution is unique.
$f()$	Density function associated with F. We assume Riemann integrability of f, i.e., f is bounded and continuous almost everywhere (Royden, 2004, pg. 85).
μ	Mean demand.