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Decision Support

An equilibrium efficiency frontier data envelopment analysis approach for evaluating decision-making units with fixed-sum outputs

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ABSTRACT

Based on the minimal reduction strategy, Yang et al. (2011) developed a fixed-sum output data envelopment analysis (FSODEA) approach to evaluate the performance of decision-making units (DMUs) with fixed-sum outputs. However, in terms of such a strategy, all DMUs compete over fixed-sum outputs with "no memory" that will result in differing efficient frontiers' evaluations. To address the problem, in this study, we propose an equilibrium efficiency frontier data envelopment analysis (EEFDEA) approach, by which all DMUs with fixed-sum outputs can be evaluated based on a common platform (or equilibrium efficient frontier). The proposed approach can be divided into two stages. Stage 1 constructs a common evaluation platform via two strategies: an extended minimal adjustment strategy and an equilibrium competition strategy. The former ensures that original efficient DMUs are still efficient, guaranteeing the existence of a common evaluation platform. The latter makes all DMUs achieve a common equilibrium efficient frontier. Then, based on the common equilibrium efficient frontier, Stage 2 evaluates all DMUs with their *original* inputs and outputs. Finally, we illustrate the proposed approach by using two numerical examples.

1. Introduction

Data envelopment analysis (DEA), firstly proposed by Charnes, Cooper, and Rhodes (1978), is a mathematical programming approach for evaluating the performance of decision-making units (DMUs) that convert multiple inputs into multiple outputs. Traditional DEA models, such as CCR (Charnes et al., 1978) and BCC (Banker, Charnes, & Cooper, 1984), divide DMUs into two groups: efficient and inefficient. Efficient DMUs construct an efficient frontier that envelops inefficient DMUs. Generally, inefficient DMUs can reduce their inputs or expand outputs freely when projecting them onto efficient frontiers for calculating their efficiencies. Traditional DEA models assume inputs and outputs are independent (Banker & Natarajan, 2011, chap. 11 of Handbook on Data Envelopment Analysis; Banker, Charnes, Cooper, & Maindiratta, 1987; Li, Yang, Chen, Dai, & Liang, 2013). However, the dependence among inputs and outputs may exist in some DEA applications. For example, Branda (2013) and Lamb and Tee (2012) described the dependence between the input (risk) and the output (return) in financial performance evaluation In contrast to this kind of dependence, this study focuses on a particular form of dependence among outputs which arises when the sum of DMUs' outputs is fixed.

In the fixed-sum outputs environment, one DMU expanding its outputs may reduce others' outputs. For instance, the market share of a certain industry is constant, so each firm in the industry attempts to compete for more market share from its competitors. Similar cases also occur in Olympic Games evaluations (Lozano, Villa, Guerrero, & Cortes, 2002; Churilov & Flitman, 2006; Li, Liang, Yao, & Hiroshi, 2008; Wu, Liang, Chen, 2009; Wu, Liang, Yang, 2009), in which the total number of medals is fixed.

When evaluating Olympic Achievements, Lins, Gomes, Soares de Mello, and Soares de Mello (2003) observed that increasing the number of medals won by one country will reduce the total number of medals won by the other countries. They proposed two strategies to accommodate the competition among various nations (viewed as DMUs) over a fixed number of medals: one is the same amount reduction strategy, which balances the increase of the outputs of the DMU under evaluation with an equal reduction in the outputs of the others. Nevertheless, the authors are conscious that the strategy will lead to negative outputs when certain DMUs' actual outputs are smaller than an equal amount reduction. This goes against the fact that all of the variables in

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traditional DEA methods must be non-negative[2] (Cooper, Seiford, & Tone, 2000). The other is a proportional reduction strategy, by which the output losses of others are proportional to their true outputs. However, the second strategy will result in that output losses rely heavily on true outputs. More specifically, once the total output augment is known, the output increase that each evaluated DMU derived from any other DMU is determined because the ratio of the true output of each DMU to the sum of outputs of all DMUs can be calculated in advance. However, in actual DMUs, output losses should be determined by free competition (Yang, Wu, Liang, & Liam, 2011) rather than subjectively being assigned by decision-makers. Moreover, Lins et al. (2003) can address only a one-dimensional fixed output problem. Multi-dimensional cases have to be pre-processed to one dimension before the model is applied.

To overcome shortfalls in Lins et al. (2003) and Yang et al. (2011) developed a new DEA model, based on a minimal reduction competition strategy, to evaluate DMUs with fixed-sum outputs. By this strategy, the minimal output augment, which the evaluated DMU needs to become technically efficient, is defined to be equal to the sum of the minimal output losses of other competitors. The authors claim that such a competition strategy is the most convenient way for performance improvement by each DMU because both free competition and the potential "opponents" are considered simultaneously. Nevertheless, this strategy may not take into account the problem that all DMUs are evaluated based on different platforms. Specifically, according to this strategy, each DMU takes part in a "no memory" competition, indicating that the results obtained in the previous-round competition are ignored when entering the next round. For example, suppose there are two DMUs, named A and B, with a single fixed-sum output. When DMU A is under evaluation, we place the original data of A and B into the Model (3) in Yang et al. (2011) and obtain the output increment that DMU A needs to be efficient. Naturally, this output increment equals the output loss of DMU B. Next, when DMU B is under evaluation, both the output augment and output loss in A are ignored, and the original data are used to measure the output increment of DMU B and the output loss of DMU A. As a result, DMU A and B are evaluated based on different efficient frontiers via the FSODEA model. In fact, the results in each "no memory" competition will construct a different platform for each DMU evaluation.

To solve this problem, we propose an equilibrium efficiency frontier data envelopment analysis (EEFDEA) approach to evaluate DMUs with fixed-sum outputs on a common platform. The approach can be divided into two stages. Stage 1 constructs a common platform (or equilibrium efficient frontier) for evaluation based on two strategies. One is an extended minimal adjustment strategy that ensures that originally efficient DMUs are still efficient. The other is an equilibrium competition strategy that makes all DMUs achieve a common equilibrium efficient frontier. Based on the common frontier, Stage 2 evaluates all DMUs with their original inputs and outputs.

The remainder of this study is organized as follows. Section 2 reviews some fixed -sum output DEA models proposed by Yang et al. (2011). Section 3 introduces the EEFDEA approach via two stages: constructing a common equilibrium efficient frontier and proposing a corresponding evaluation model. Section 4 uses two numerical examples to verify and illustrate the EEFDEA approach. Conclusions and directions for future research are provided in Section 5.

[2] In practice, outputs might be negative, such as negative profits of companies. Some scholars have attempted to deal with such the negative data (Kerstens & Woestyne, 2011; Silva Portela, Thanassoulis, & Simpson, 2004). However, the negative data may not be allowed in some of the fixed-sum-output cases, since it might make no sense in practice. For example, the number of medals obtained by participating countries in Olympics and the market share of one company cannot be negative, but sums of medals and market shares are fixed.

2. Preliminaries

Suppose there are n DMUs and that each $DMU_j (j = 1, 2, \ldots, n)$ consumes m inputs $x_{ij} (i = 1, \ldots, m)$ to produce s variable-sum outputs $y_{rj} (r = 1, \ldots, s)$, and l fixed-sum outputs $f_{tj} (t = 1, \ldots, l)$. Evidently, variable-sum outputs are those whose sum can be expandable. Also, fixed-sum outputs satisfy constraints $\sum_{j=1}^{n} f_{tj} = F_t, \forall t \in (1, \ldots, l)$, where F_t is a constant. The traditional BCC model is formulated to access the efficiency of $DMU_k (k = 1, 2, \ldots, n)$ as follows:

$$e_k = Max\ \frac{\sum_{r=1}^{s} u_r y_{rk} + \sum_{t=1}^{l} w_t f_{tk} + \mu_0}{\sum_{i=1}^{m} v_i x_{ik}}$$
$$s.t.\ \frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{t=1}^{l} w_t f_{tj} + \mu_0}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1 \quad \forall j \quad u_r, w_t, v_i \geqslant 0, \forall r, t, i \tag{1}$$

where u_r, w_t, v_i are multipliers of the rth variable-sum output, the tth fixed-sum output and the ith input, respectively. Note that the Model (1) changes to be the CCR model when the variable μ_0 equals zero. In order to enable easy solution, the Model (1) can be transformed into a linear program by Charnes–Cooper transformation (Charnes & Cooper, 1962) as follows:

$$e_k = Max \sum_{r=1}^{s} \mu_r y_{rk} + \sum_{t=1}^{l} \omega_t f_{tk} + u_0$$
$$s.t.\ \sum_{r=1}^{s} \mu_r y_{rj} + \sum_{t=1}^{l} \omega_t f_{tj} - \sum_{i=1}^{m} \upsilon_i x_{ij} + u_0 \leqslant 1 \quad \forall j \tag{2}$$
$$\sum_{i=1}^{m} \upsilon_i x_{ik} = 1 \qquad \mu_r, \omega_t, \upsilon_i \geqslant 0, \forall r, t, i$$

Suppose an optimal solution of the Model (2) is $(e_k^*, \mu^*, \omega^*, \upsilon^*)$, where $\mu^* = (\mu_1^*, \mu_2^*, \ldots, \mu_s^*), \omega^* = (\omega_1^*, \omega_2^*, \ldots, \omega_l^*)$ and $\upsilon^* = (\upsilon_1^*, \upsilon_2^*, \ldots, \upsilon_m^*)$. Then DMU_k is efficient if $e_k^* = 1$ and there exists at least one optimal solution $(\mu^*, \omega^*, \upsilon^*)$, with $\mu^* > 0$ $\omega^* > 0$ and $\upsilon^* > 0$. Otherwise, it is inefficient (Cooper et al., 2000).

Model (2) can evaluate the efficiency of each DMU in many situations but fails to assess DMUs with fixed-sum outputs. Considering competition over limited resource among DMUs, Yang et al. (2011) proposed the following DEA model, based on a minimal reduction strategy for calculating the minimal aggregated output expansion size for DMU_k:

$$Min \sum_{t=1}^{l} w_t \alpha_{tk}$$
$$s.t.\ \frac{\sum_{r=1}^{s} u_r y_{rk} + \sum_{t=1}^{l} w_t (f_{tk} + \alpha_{tk}) + \mu_0}{\sum_{i=1}^{m} v_i x_{ik}} = 1 \quad \text{(3a)}$$
$$\frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{t=1}^{l} w_t (f_{tj} - \delta_{tj}) + \mu_0}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1 \quad \forall j \text{ and } j \neq k \quad \text{(3b)}$$
$$\sum_{\substack{j=1 \\ j \neq k}}^{n} \delta_{tj} = \alpha_{tk} \quad 0 \leqslant \delta_{tj} \leqslant f_{tj}\ \forall t, \quad \forall j \text{ and } j \neq k \quad \text{(3c)}$$
$$u_r, w_t, v_i \geqslant 0, \forall r, t, i, \quad \mu_0 \text{ is free} \tag{3}$$

In the Model (3),[3] α_{tk} is the output increment that DMU_k derives from others and δ_{tj} is the output reduction of $DMU_j (j \neq k)$ as an offset for the output increase. The objective function of the Model (3) is to

[3] Note that the Model (3) in this paper is the same as the Model (3) in Yang et al. (2011). And we find that their model cannot transform to be the linear the Model (11) in appendix of Yang et al. (2011) unless we replace the objective function of the Model (3) $Min \sum_t u_t \beta_{rk}$ with. $Min \sum_r u_t \beta_{rk} / \sum_i v_i x_{ik}$. This mistake also gives rise to incorrect results in numerical examples of their paper.