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Interfaces with Other Disciplines

All-pay auctions with pre- and post-bidding options

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ABSTRACT

Motivated by the emergence of online penny or pay-to-bid auctions, in this study, we analyze the operational consequences of all-pay auctions competing with fixed list price stores. In all-pay auctions, bidders place bids, and highest bidder wins. Depending on the auction format, the winner pays either the amount of their bid or that of the second-highest bid. All losing bidders forfeit their bids, regardless of the auction format. Bidders may visit the store, both before and after bidding, and buy the item at the fixed list price. In a modified version, we consider a setting where bidders can use their sunk bid as a credit towards buying the item from the auctioneer at a fixed price (different from the list price). We characterize a symmetric equilibrium in the bidding/buying strategy and derive optimal list prices for both the seller and auctioneer to maximize expected revenue. We consider two situations: (1) one firm operating both channels (i.e. fixed list price store and all-pay auction), and (2) two competing firms, each operating one of the two channels.

1. Introduction

Auctions are fascinating and important sales mechanisms that date back to antiquity. They have been used extensively in both *business-to-consumer* and *business-to-business* markets. With the advent of the Internet, auctions have also become popular in *consumer-to-consumer* markets as exemplified by the online auction behemoth eBay. In addition to the many Internet-based traditional auction formats, where a seller auctions an item to a group of buyers, the dot-com entrepreneurial spirit gave rise to many non-traditional auction formats and auction-based business solutions. Two notable examples include Priceline, a *Name-Your-Own-Price* 'reverse auction' mechanism, and Google's AdWords, a keyword auction for online ad space.

The latest Internet-based auction mechanism that has been growing in popularity is the *penny* or *pay-to-bid* auction, examples of which include quibids.com, dealdash.com, beezid.com, and the former online auction sites swoopo.com and offandaway.com. Three particular features separate these online auctions sites from the more common ones and, in particular, from eBay. First, unlike eBay, which *hosts* auctions for sellers only, the penny auction sites *act as the seller* of the items auctioned, which are usually brand new and are predominately from the consumer electronics category. Second, unlike the hard-ending rule of eBay, where an auction ends at a pre-specified date and time, penny auctions employ a *'going, going,. . .gone'* ending rule by extending the auction clock for each incoming bid in the final moments. Third, the penny auction mechanism is drastically different in that bidders first buy a pack of 'bids' for a fixed price (e.g., a pack of 100 'bids' for $60), and these 'bids' are then used to nominally raise the price in an auction (e.g., by one cent — hence the name *penny auction*). In other words, for the given example, raising the auction price by one cent costs the bidder 60 cents. The bidder who places the last 'bid' wins the item and pays the final auction price, in addition to the incurred sunk cost of the 'bids' purchased at the outset of the process; all non-winning bidders incur only the sunk cost of 'bidding'. Despite their peculiar format, due to the nature of this sunk bidding cost, penny auctions can, in effect, be characterized as *second-price all-pay* auctions.

All-pay auctions are similar to traditional auctions in that whoever submits the highest bid wins the item; however, unlike traditional auctions, where non-winning bidders pay nothing, in all-pay auctions, non-winning bidders forfeit their bids (i.e., all bidders pay their bid but only the highest bidder wins the item). The added prefix *second-price* refers to the fact that the highest bidder must pay the amount of the second-highest bid only, and not the actual amount they bid. Although their process might seem a bit peculiar, all-pay auctions are well established in the auction literature. More details regarding auction theory and all-pay auctions are discussed in Section 2. To see how penny auctions constitute, in effect,

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second-price all-pay auctions, consider the case when there are only two bidders. With only two bidders, the first bidder who fails to counter-bid will lose and forfeit the sunk bidding cost, while the winning bidder will have matched the cost of bidding (i.e., paid the same amount as the second-highest bidder). Of course, the winner must also pay the additional final auction price, but this amount will be an order of magnitude smaller than the sunk bidding cost, and thus, can be considered potentially negligible. The extension to multiple bidders is immediate: in each "bidding round", bidders must decide whether to stay in the game by placing a bid, or decide to drop out and accept the sunk cost of bidding. There remains, however, the issue how to enforce all bidders to participate in each "bidding round". More details regarding the link between penny auctions and all-pay auctions are discussed in Section 3.

Bidders are drawn to participate in online penny auctions by the chance of winning an item at a highly discounted price. From the online penny auctioneers' perspective, the expectation is that there will be enough active bidders such that the revenue from all 'bids' exceeds the cost of the item. Typical anecdotal stories tell us that a winning bidder may acquire a brand new smartphone for just $50, while the penny auction site collects a total revenue of $2000 from the bidding process. These stories have generated much negative press in academia, newspapers and blogs, and critics have basically labeled penny auctions as online gambling (Robinson, Giebelhausen, & Cotte, 2013; USA Today, 2011; NY Times, 2009a, 2009b; Guina, 2009). In an extreme example, a class-action lawsuit has been filed against quibids.com, claiming that it is more akin to a gambling website than an auction website (BusinessWire, 2010). Since online gambling is regulated in most countries, the general strategy from the penny auction sites has been to position themselves as 'entertainment shopping' channels. Due to the unusual and unintuitive format of all-pay auctions it is perhaps not surprising that penny auction websites are being labeled as online gambling sites.

The main motivation of this paper is to clarify some of the misunderstandings around penny auction sites, as well as to provide some insights regarding bidding strategies and analyze the operational consequences for sites themselves. Specifically, we consider a setting where bidders have access to two sales channels: a store that sells items at a fixed list price, and an all-pay auction. Bidders who choose to bid in the all-pay auction take into consideration that they can, either before or after bidding, buy the item directly at the fixed list price from the store. In a modified version to the all-pay auction, we consider a format where all losing bidders are given the option of using their sunk bid as a credit towards purchasing the item from the auctioneer at a fixed price (which, naturally, is higher than the store list price). The feature of using the sunk bid as a credit constitutes an interesting aspect of real-world penny auction sites because it provides an additional twist compared to traditional auctions. Our main research objectives involve uncovering whether, how, and for whom the added auction channel provides value, and analyzing what effects competing auction channels have on setting optimal list prices. To analyze the dynamics between the seller and auctioneer, we consider two cases. In the first case, we assume there is only one firm operating both the fixed list price store and the all-pay auction. In the second case, we assume there are two competing firms: one operating the fixed list price store and one operating the all-pay auction.

The main contributions of our paper are as follows. First, we illuminate the dynamics of online penny auctions as an application of second-price all-pay auctions. In contrast to the negative publicity and previous published results regarding penny auctions, our analysis indicates that consumers are in fact better off at the expense of the penny auction site. Second, we develop equilibrium bidding strategies in a private valuation framework for both first- and second-price all-pay auctions when bidders have pre- and post-bidding options to consider. Our inclusion of posted prices in the all-pay auction framework and with the addition of using lost bids as credits towards posted-price purchasing is, to the best of our knowledge, a novel extension. Third, we develop optimal posted pricing policies for the seller and auctioneer and discuss the economic implications. Finally, we illustrate the effect of 'sunk-bid credit' on the distribution of both final auction price and total revenue. For readability, all proofs are relegated to the Appendix.

2. Auction theory background

In addition to their commercial and social aspects, auctions are interesting from a theoretical perspective. The voluminous literature on auctions and bidding mechanisms spans not only Operations Research and Economics, but also Information Systems, Marketing, Computer Science, Statistics, and Theoretical Biology. However, despite the wide proliferation of auction theory, critiques regarding limitations and overly restrictive assumptions have also been raised. A motivating example appears in the almost exclusive consideration of analyzing auctions in isolation and independent of overall context, an issue raised by Rothkopf and Harstad (1994), Pinker, Seidmann, and Vakrat (2003), Shen and Su (2007) and Haruvy et al. (2008). Rarely, if ever, do bidders lack external options when deciding whether to bid and how much to bid in an auction. This research expands the traditional auction framework and analyzes the dynamics when bidders consider external options, both before and after bidding. To establish theoretical context, we begin with a brief discourse on fundamental auction theory.

The standard private valuation, single-item auction model centers on a fixed number of bidders N, each with *i.i.d.* valuation V drawn from distribution $F_V(v) = \Pr\{V \leqslant v\}$, with support on $[\underline{v}, \bar{v}]$. Each bidder knows only their own (realized) valuation v, the number of bidders N, and the distribution $F_V(v)$, but not the realized valuations of the other $N-1$ bidders. Although auction formats may either be *open* or *sealed-bid*, in this paper, we focus on the sealed-bid versions. In sealed-bid auctions, each bidder places their bid in a sealed envelope. When all bids have been submitted, the auctioneer opens the bids and announces, according to the specified format, the winner and the amount the winner has to pay. Typically two auction formats are considered: *first-price* auction, where the highest bidder wins and pays the amount of their bid, and *second-price* auction, where the highest bidder wins but pays the amount of the second highest bid. A third and slightly more unusual format is the aforementioned *all-pay* auction, which comes in two general versions: the *first-price all-pay* auction, where everyone pays the amount they bid, but only the highest bidder wins the item; and the *second-price all-pay* auction, where all losing bidders pay their bid while the highest bidder, who wins the item, pays the amount of the second-highest bid.[1] For all four auction formats, a bidder's decision of whether to bid and the bid amount b can be illustrated by a decision tree. See the decision tree in Fig. 1, where x_b is the price the bidder pays upon winning the auction, and y_b is the cost the bidder incurs for losing the auction. Note that x_b and y_b are functions of the bid amount b. In a first-price and second-price auction, $y_b = 0$, while for the two all-pay auctions $y_b = b$. In a first-price and first-price all-pay auction, $x_b = b$, while in a second-price and second-price all-pay auction, x_b is the amount of the second highest bid. The probability of winning ρ_b depends on the bid b.

[1] Based on applications from theoretical biology, second-price all-pay auctions are often referred to as *war of attrition*. In this paper, we use the term 'second-price all-pay auction'.