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Continuous Optimization

The multicriteria p-facility median location problem on networks

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ABSTRACT

In this paper we discuss the multicriteria p-facility median location problem on networks with positive and negative weights. We assume that the demand is located at the nodes and can be different for each criterion under consideration. The goal is to obtain the set of Pareto-optimal locations in the graph and the corresponding set of non-dominated objective values. To that end, we first characterize the linearity domains of the distance functions on the graph and compute the image of each linearity domain in the objective space. The lower envelope of a transformation of all these images then gives us the set of all non-dominated points in the objective space and its preimage corresponds to the set of all Pareto-optimal solutions on the graph. For the bicriteria 2-facility case we present a low order polynomial time algorithm. Also for the general case we propose an efficient algorithm, which is polynomial if the number of facilities and criteria is fixed.

1. Introduction

Many real-world applications are concerned with finding an optimal location for one or more new facilities on a network (road network, power grid, etc.) minimizing a function of the distances between these facilities and a given set of existing facilities (clients, demand points), where the latter typically coincide with vertices. For a recent book on location theory the reader is referred to Nickel and Puerto (2005) and references therein. Since the first seminal paper by Hakimi (1964), an ever growing number of results have been published in this field.

The majority of research focuses on the minimization of a single objective function that is increasing with distance. However, in the process of locating a new facility usually more than one decision maker is involved. This is due to the fact that often the cost incurred with the decision is relatively high. Furthermore, different decision makers may (or will) have different (conflicting) objectives. In other situations, different scenarios must be compared due to uncertainty of data or still undecided parameters of the model. One way to deal with these situations is to apply scenario analysis. Another way of reflecting uncertainty in the parameters is to consider different replications of the objective function. Hence, there exists a large number of real-world problems which can only be modeled suitably through a multicriteria approach, especially when locating public facilities.

An additional difficulty is that we are usually dealing with conflicting criteria and a single optimal solution does not always exist (which would be an optimal solution for each of the criteria). Therefore, an alternative solution concept has to be used. One possibility is to determine the set of non-dominated solutions. That is, solutions such that there exists no other solution which is at least as good for all decision makers and strictly better for at least one of them. These solutions are often called Pareto-optimal. For an overview on multicriteria location problems the reader is referred to Nickel, Puerto, and Rodríguez-Chía (2005).

In contrast to the practical needs described above, network location research involving multiple criteria has received little attention, especially when it comes to multiple facilities. In this paper, we consider the p-facility median location problem with several objective functions. Hereby, each objective function is representing the goal of one decision maker and the aim is to locate p facilities in order to minimize the total weighted distance from the clients to their closest facility. The weights assigned to clients vary from one decision maker to another, yielding different objective functions. It might even happen that one of the facilities is desirable for some decision makers and, at the same time, undesirable for others. Undesirable facilities are usually modeled using negative weights. See Eiselt and Laporte (1995) for more details on these problems. Before we discuss the literature, we present a practical example for this model. Suppose we want to locate two garbage dumps and we have a set of residential and recreational

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areas and a set of industrial sites where garbage has to be collected. There are two decision makers involved: the "Business economist" who has to keep the costs in check and the "Politician" who is concerned about the nuisance of the garbage dumps and the garbage trucks on the population. The business economist wants the dumps to be close to all sites to minimize travel times and costs. To that end he associates positive weights with the residential and industrial areas that are proportional to the average number of required garbage collections. In contrast to that, the politician wants to minimize the nuisance of the garbage dumps and of the trucks frequenting the garbage dumps for the population. Therefore, he assigns to each site a second, negative value. The smaller the weight is, the more likely it is that the residential area is far away from the dumps and the less likely it is that trucks that are not bound for these areas are simply passing through them on their way to the dumps. Formulating this problem in mathematical terms results in a bi-criteria 2-facility location model.

There are many other applications of multicriteria multifacility location problems. Bitran and Lawrence (1980) consider the multicriteria location of regional service offices in the expanding operating territories of a large property and liability insurer. These offices serve as first line administrative centers for sales support and claims processing. Another application of multiobjective optimization in the context of location theory can be found in Johnson (2001) that discusses a spatial decision making problem for housing mobility planning. Ehrgott and Rau (1999) present an analysis of a part of the distribution system of BASF SE, which involves the construction of warehouses at various locations. The authors evaluate 14 different scenarios and each of these scenarios is evaluated with the minimal cost solution obtained through linear programming and the resulting average delivery time at this particular solution. For more applications see Schöbel (2005), Carrano, Takahashi, Finseca, and Neto (2007), and Kolokolov and Zaozerskaya (2013).

Concerning the methodological aspects of multicriteria network location problems, Hamacher, Labbé, and Nickel (1999) discuss the network 1-facility problem with median objective functions. They show that for Pareto-optimal locations on undirected networks no node dominance result can be proven. Hamacher, Labbé, Nickel, and Skriver (2002) provide a polynomial time algorithm for the 1-facility problem when the objectives are both weighted median and anti-median functions. The method is generalized for any piecewise linear objective function. Zhang and Melachrinoudis (2001) develop a polynomial algorithm for the 2-criteria 1-facility network location problem maximizing the minimum weighted distance from the service facility to the nodes (maximin) and maximizing the sum of weighted distances between the service facility and the nodes (maxisum). Skriver, Andersen, and Holmberg (2004) introduce two sum objectives and criteria dependent edge lengths for the 1-facility 2-criteria problem. Nickel and Puerto (2005) solve the 1-facility problem when all objective functions are ordered medians. Colebrook and Sicilia (2007a, 2007b) provide polynomial algorithms for solving the cent-dian 1-facility location problem on networks with criteria dependent edge lengths and facilities being attractive or obnoxious.

Concerning the single criterion multifacility location problem on networks, Kalcsics (2011) derives a finite domination set for the p-median problem with positive and negative weights. For the 2-facility case, the author presents an efficient solution procedure using planar arrangements. Based on this approach, Kalcsics, Nickel, Puerto, and Rodríguez-Chía (2012) solve the 2-facility case for different equity measures.

Many of the previous papers study the problem on trees as a particular case of generalized networks. The first work dealing with several objectives and facilities is provided by Tansel, Francis, and Lowe (1982) who develop an algorithm for finding the efficient frontier of the biobjective multifacility minimax location problem on a tree network. This problem involves as objective functions the maximum of the weighted distances between specified pairs of new and existing facilities.

Despite its intrinsic interest as discussed above, to the best of our knowledge there are no papers discussing the multicriteria p-facility median location problem on networks and no results are known until the moment to obtain the set of Pareto-optimal solutions.

The remainder of this paper is organized as follows. Section 2 introduces the notation and concepts used throughout the paper. Section 3 presents some properties of the k-criteria p-facility median problem on networks. Section 4 is devoted to the development of a polynomial algorithm for the 2-criteria 2-facility version of the problem. A solution procedure for the general case is proposed in Section 5. Finally, Section 6 contains some conclusions and possible extensions of the analyzed problems.

2. Problem description and general concepts

2.1. Problem definition

Let $G=(V,E)$ be an undirected connected graph with node set $V=\{v_1,\dots,v_n\}$ and edge set $E=\{e_1,\dots,e_m\}$. Each edge $e\in E$ has a positive length $\ell(e)$, and is assumed to be rectifiable. Let $A(G)$ denote the continuum set of points on edges of G. We denote a point $x\in e=[u,v]$ as a pair $x=(e,t)$, where t $(0\leqslant t\leqslant 1)$ gives the relative distance of x from node u along edge e. For the sake of readability, we identify $A(G)$ with G and $A(e)$ with e for $e\in E$. Let $k\geqslant 1$ be the number of criteria of the problem and define $Q=\{1,\dots,k\}$. Each vertex $v_i\in V$ has a real-valued weight $w_i^q\in\mathbb{R}$, $q\in Q$. Let $J=\{1,\dots,p\}$, where p is the number of facilities to be located. We denote by $X=(x_1,\dots,x_p)$ the vector of locations of the facilities, where $x_j\in G$, $j\in J$. (Note that in order to allow co-location, which is quite common in location problems with negative weights, we have to represent the facility locations using a vector.) In the remainder, we use the notions location vector and solution synonymously.

We denote by $d(x,y)$ the length of the shortest path connecting two points $x,y\in G$. Let $v_i\in V$ and $x=([v_r,v_s],t)\in G$. The distance from v_i to x entering the edge $[v_r,v_s]$ through v_r (v_s) is given as $D_i^+(x)=d(v_r,x)+d(v_r,v_i)$ ($D_i^-(x)=d(v_s,x)+d(v_s,v_i)$). Hence, the length of a shortest path from v_i to x is given by $D_i(x)=\min\{D_i^+(x),D_i^-(x)\}$. As $d(v_r,x)=t\cdot\ell(e)$ and $d(v_s,x)=(1-t)\cdot\ell(e)$, the functions $D_i^+(x)$ and $D_i^-(x)$ are linear in x and $D_i(x)$ is piecewise linear and concave in x, cf. Drezner (1995). The distance from v_i to its closest facility is finally defined as $D_i(X)=\min_{j\in J}D_i(x_j)=\min_{j\in J}\{D_i^+(x_j),D_i^-(x_j)\}$. In the following, we call the functions $D_i^{+/-}(x)$ and $D_i(X)$ distance functions of node v_i. Moreover, we say that $D_i^a(x_j)$, $a\in\{+,-\}$, is active for X, if $D_i^a(x_j)=D_i(X)$.

We consider the objective function $F(X)=(F^1(X),\dots,F^{|Q|}(X))$, where each $F^q(X)$, $q\in Q$, is a median function defined as:

$$F^q(X)=\sum_{i\in V}w_i^q\,D_i(X).$$

We assume the usual definition of Pareto-optimality or efficiency (Ehrgott, 2005). That is, a solution X is called efficient or Pareto-optimal, if there exists no solution X' which is at least as good as X with respect to all objective function values and strictly better for at least one value, i.e., $\nexists X':F_q(X')\leqslant F_q(X),\ \forall q\in Q$, and $\exists q\in Q:F_q(X')<F_q(X)$. If X is Pareto-optimal, $F(X)\in\mathbb{R}^k$ will be called a non-dominated point. If $F_q(X)\leqslant F_q(X')\ \ \forall q\in Q$ and $\exists q\in Q:F_q(X)<F_q(X')$ we say X dominates X' in the decision space and $F(X)$ dominates $F(X')$ in the objective space.