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Continuous Optimization

# The self regulation problem as an inexact steepest descent method for multicriteria optimization

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ABSTRACT

In this paper we study an inexact steepest descent method for multicriteria optimization whose step-size comes with Armijo's rule. We show that this method is well-defined. Moreover, by assuming the quasi-convexity of the multicriteria function, we prove full convergence of any generated sequence to a Pareto critical point. As an application, we offer a model for the Psychology's self regulation problem, using a recent variational rationality approach.



## 1. Introduction

The method described by (Burachik, Graña Drummond, Iusem, & Svaiter, 1995) for a continuously differentiable optimization problem generates a sequence with the property that any accumulation point of it is a critical point for the objective function. It is the steepest descent method with Armijo's rule, which was later generalized by (Fliege & Svaiter, 2000) for multicriteria optimization, in the case where the objective function is a vectorial function. The result of full convergence is assured when the objective function is convex, provided that the problem's solution set is non-empty, see (Burachik et al., 1995), or in a more general way, when the objective function is quasi-convex; see (Bello Cruz & Lucambio Pérez, 2010; Kiwiel & Murty, 1996). Graña Drummond and Svaiter (2005) generalized this result for convex vectorial optimization and (Bento, Ferreira, & Oliveira, 2012) generalized it for quasi-convex multicriteria optimization, see also (Bello Cruz, Lucambio Pérez, & Melo, 2011). See (Miglierina, Molho, & Recchioni, 2008) for a computational approach of a gradient like method in the context multiobjective. For extensions of other scalar optimization methods to the vectorial setting, see (Bonnel, Iusem, & Svaiter, 2005; Ceng, Mordukhovich, & Yao, 2010; Fliege, Graña Drummond, & Svaiter, 2009; Villacorta & Oliveira, 2011) and references therein.

As far as we know, (Bento et al., 2012) were the first ones who presented a result of full convergence of the exact steepest descent method, with Armijo's rule, for quasi-convex multicriteria optimization – a work that includes contributions in both Euclidean and Riemannian contexts. In the present paper, we study the method proposed by (Fliege & Svaiter, 2000), which is the inexact version of the method presented in (Bento et al., 2012). Relative errors on the search directions are admitted in this method, that is, an approximation of the exact search direction is computed at each iteration. Specifically, we present the global convergence of any sequence generated by this method to a Pareto critical point (resp. weak Pareto optimal point) of the multiobjective optimization problem in the quasiconvex case (resp. pseudo-convex case).

This paper is organized as follows: In Section 2, we present the self regulation problem under the context of Psychology; In Section 3, the multicriteria problem and the first order optimality condition for it are presented, along with some basic definitions; In Section 4, the inexact steepest descent method used to find a solution for multicriteria problems is stated and the well-definition of the sequence generated by it is established; In Section 5, a result for partial convergence of the method is presented without any additional assumption on the objective function. Moreover, assuming that the objective function is quasi-convex and that the Riemannian manifold has non-negative sectional curvature, we present a result for full convergence; Finally, Section 6 offers a "distal-proximal" model of self regulation in Psychology, using a recent variational rationality approach; see (Soubeyran, 2009 – Variational rationality, a theory of individual stability

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and change: worthwhile and ambidextry behaviors), (Soubeyran, 2010 – Variational rationality and the unsatisfied man: routines and the course pursuit between aspirations, capabilities and beliefs), (Soubeyran, 2012a – Worthwhile to change course pursuits and behavioral traps) and (Soubeyran, 2012b – Variational rationality: a course pursuit between desired ends and feasible means), which models behaviors as an approaching or avoidance process, a course pursuit between "desired enough" ends and "feasible enough" means.

## 2. The self regulation problem

In this section, devoted to applications, we will focus our attention to the so-called "multiple goals self regulation problem", in the field of behavioral sciences. Our paper extends the steepest descent method of (Fliege & Svaiter, 2000) to the quasi-convex case in multicriteria optimization. We will show a strong link between our research and the "variational rationality" approach of Soubeyran's "theories of change" (Soubeyran, 2009, 2010, 2012a, 2012b). Change problems consider "why, how, and when" it is worthwhile to move from a bad or not so favorable situation $x \in X$ to a better one $y \in X$ that could be known or unknown. There is a number of different formulations that depend on the context, but in general the limit case of full rationality is an optimizing one, and the case of bounded rationality is a better one.

The variational rationality approach examines two kinds of "change problems": (i) adaptive choice problems, such as the problem of "selecting the context to choose from", that is, the formation of sets to be taken in consideration; (ii) transformation problems, such as creation and destruction, invention and innovation, or problems involving evolution of institutions, dynamic interactions, and changes of different nature, as health, behavioral, organizational and cultural. It has its applications in Economics, Decision Theory, Management, Psychology, Artificial Intelligence, Philosophy, Sociology, and Applied Mathematics (e.g., Variational Analysis, Optimization and Variational Inequalities). In this variational context, our present paper shows how useful it is to set together distal and proximal goals in order to reach a distal goal. This variational approach emphasizes, each step of the process, two main variational principles: a "satisfactory, but not too much sacrificial" principle and a "worthwhile to change" principle. Since the space of situations is the Euclidian space $X = \mathbb{R}^n$, changes $u = y - x$ from a given situation $x$ to a hopefully better situation $y$ can be characterized by their directions $v \in X$ and their depth $t > 0$. In the context where $u = tv$, these two variational principles deals with:

(i) the choice, for each step, of a "satisfactory – but not too much sacrificing" direction (that is, a directional "satisfactory – but not too much sacrificing" principle);
(ii) the choice, for each step, of a "worthwhile change" principle.

### 2.1. Self regulation problems

The notions of self regulation and goal are discussed now. Self regulation considers the systematic activities or efforts that are made to direct thoughts, feelings, and actions towards the accomplishment of someone's goals (Zimmerman, 2000). A goal is a conscious or unconscious mental representation of some future objective (to approach or to avoid), and it can be distal or proximal, abstract or concrete, vague or precise, long term or short term, extrinsic or intrinsic, high or low in commitment, more or less desirable, and more or less feasible. Related to its feasibility aspects we list importance, priority, urgency, direction, intensity, difficulty, and measurability.

Self regulation has two aspects, a positive and a negative one. The positive side of it considers purposive processes where agents are engaged in goal-directed actions. It examines goal setting, goal striving and goal pursuit processes.

(a) Goal setting is the mental process of moving from the consideration of distal goals to the formation of more proximal goals. Distal goals are desired future ends (visions, futures plans, etc.), either promotion aspirations (such as ideals, fantasies, dreams, wishes, hopes or challenges) or prevention aspirations (as duties and obligations). They represent desirable but quite unrealistic and vague ends (higher order goals). Proximal goals can be wants, intentions, task goals, i.e. much more feasible but less desirable intermediate ends (sub goals).
(b) Goal striving (goal implementation) examines the transition phase between setting a distal goal and reaching it.
(c) Goal pursuit (goal revision) focuses on the final phase, after reaching the given goal or failing to reach it. It examines the role of feedbacks (self evaluations of successes and failures, including the revision of causal attributions and self efficacy beliefs; see (Tolli & Schmidt, 2008)) in order to revise goals.

Our paper considers only the positive aspect of self regulation. It focuses on proximal goal setting activities, examines some aspects of goal revision activities, and refuses to consider goal striving activities.

### 2.2. Setting proximal goals

#### 2.2.1. The Michael Jordan "step by step" principle

The famous basketball player Michael Jordan wrote the following about goal setting in his book (Jordan & Miller, 1994), "I approach everything step by step .... I had always set short-term goals. As I look back, each one of the steps or successes led to the next one. When I got cut from the varsity team as a sophomore in high school, I learned something. I knew I never wanted to feel that bad again .... So I set a goal of becoming a starter on the varsity. That's what I focused on all summer. When I worked on my game, that's what I thought about. When it happened, I set another goal, a reasonable, manageable goal that I could realistically achieve if I worked hard enough .... I guess I approached it with the end in mind. I knew exactly where I wanted to go, and I focused on getting there. As I reached those goals, they built on one another. I gained a little confidence every time I came through ...".

#### 2.2.2. Goal hierarchies and goal proximity: the Bandura dual "proximal–distal" self regulation principle:

Bandura (1997) argued that people possess multiple systems of goals, hierarchically arranged from proximal goals to extreme distal goals. Goal proximity defines "how far goals are conceptualized into the future". A goal hierarchy interconnects at least three levels of goals: peak goals (higher order goals, such as visions, dreams, fantasies, aspirations, ideals, wishes, hopes), distal goals (challenges, wants, intentions), and task goals. A subset of task goals can be subordinate to distal goals which can be subordinate to peak goals. Hence, the proximal goal distinction is relative to the interconnected network of goals. Other goal is providing the temporal context. The main point to be emphasized here is that distal goals and proximal goals are used for different and complementary functions connected to cognition, affection, motivation and conation related to goal difficulty, goal commitment and psychological distance.

(i) distal goals define desired ends (enduring aspirations) that attract individuals;