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Exact and approximation methods for dependability assessment of tram systems with time window

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ABSTRACT

The transportation system examined in this paper is the city tram one, where failed trams are replaced by reliable spare ones. If failed tram is repaired and delivered, then it comes back on work. There is the time window that failed tram has to be either replaced (exchanged) by spare or by repaired and delivered within. Time window is therefore paramount to user perception of transport system unreliability. Time between two subsequent failures, exchange time, and repair together with delivery time, respectively, are described by random variables A, E, and D. $A|E|D$ is selected as the notation for these random variables. There is a finite number of spare trams. Delivery time does not depend on the number of repair facilities. Hence, repair and delivery process can be treated as one with infinite number of facilities. Undesirable event called hazard is the event: neither the replacement nor the delivery has been completed in the time window. The goal of the paper is to find the following relationships: hazard probability of the tram system and mean hazard time as functions of number of spare trams. For systems with exponential time between failures, Weibull exchange and exponential delivery (so M/W/M in the proposed notation) two accurate solutions have been found. For systems with Weibull time between failures with shape in the range from 0.9 to 1.1, Weibull exchange and exponential delivery (i.e. W/W/M) a method yielding small errors has been provided. For the most general and difficult case in which all the random variables conform to Weibull distribution (W/W/W) a method returning moderate errors has been given.

1. Introduction

The paper is focused on problem of system dependability. As we know, in system engineering, dependability is a measure of a system's availability, reliability, and its maintenance support. According to standard (IEC 60050-191, 1990, chap. 191): "dependability is the collective term used to describe the availability performance and its influencing factors: reliability performance, maintainability performance and maintenance support performance". Performance of transportation system is assessed from the point of view of the client/passenger. The customer requirements are in logistic theory defined by 7 R's (Ballou, 2004); the most important parameter is right time of delivery/execution of transport service. And if any failure occurs in a system, we may have time to make a corrective maintenance. Thus, the paper is focused on the approach of delay time maintenance methods and models.

Transportation system is a system in which material objects are moved in time and space. Thus, the function of transportation is to execute the movement of people and goods from one place to another in a safe and efficient way with minimum negative impact on the environment (Fricker & Whitford, 2004).

According to (Fricker & Whitford, 2004; Kutz, 2004), the transportation system is very complex with different functional characteristics depending on medium of movement, particular technology used and demand for movement in the particular medium. Aspects of these modes are e.g. vehicle, the way, control of the system, the technology of motion, intermodal transfer points, payload, drivers and pilots.

Each particular mode has a set of functional and operational characteristics. Thus, modelling a transportation system is mostly focused on identification of a structure and characteristics of structure elements (Ambroziak & Pyza, 2008). Moreover, any technical system (also transportation system) in order to successfully accomplish its intended mission must rely on effective logistic support that will be available when required. When logistic activity is narrowed down to the supply activity, we can say that the basic elements are focused on providing the necessary supplies

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(especially spare parts) and services on the proper time for the right money to provide the means to obtain a set of operational requirements. Thus, reliability/dependability modelling process of transportation system performance needs the existence of various problems connected with e.g. possible vehicle/spare parts unreliability recognizing and analyzing.

The transportation modelling problems or transportation systems dependability/maintenance issues have gained much interest since 1980s (Młyńczak, Nowakowski, Restel, & Werbińska-Wojciechowska, 2010). The basic review in the area of maintenance modelling is prepared by Pierskalla and Voelker (1976), where authors investigated discrete time vs. continuous time maintenance models, later updated by Valdez-Flores and Feldman (1989). For other surveys see e.g. (Cho & Parlar, 1991; Nakagawa, 1984; Nicolai & Dekker, 2006; Pham & Wang, 1996; Sherif, 1982; Wang, 2002).

Problem analyzed in the paper is as follows. In the city tram transportation system, failed trams are replaced by reliable spare ones. If failed tram is repaired and delivered, then it comes back on work. There is the time window that failed tram has to be either replaced (exchanged) by spare or repaired and delivered within. Time window is therefore paramount to user perception of transport system unreliability. Time between two subsequent failures, exchange time, and repair together with delivery time, respectively, are described by random variables *A*, *E*, and *D*. *A*|*E*|*D* is selected as the notation for tram system model. There is a finite number of spare trams. Delivery time does not depend on the number of repair facilities. Hence, repair and delivery process can be treated as one with infinite number of facilities. Undesirable event called hazard is the event: neither the replacement nor the repair has been completed in the time window. The goal of the paper is to find the following relationships: hazard probability of the tram system and mean hazard time as functions of number of spare trams when at least one random variable from the *A*, *E*, and *D* is not exponential.

Similar approach as presented in the paper can be applied for tram system models when repair and delivery process is executed by finite amount of repair facilities (repair teams). Such generalizations are presented in Section 8.2.

These relationships are relatively easy to find while assuming exponential distributions of analyzed random variables. But many years of experience on the field of means of transport reliability testing (Jodejko-Pietruczuk & Molecki, 2008; Nowakowski, 1999) shows inadequacy of that assumptions (Table 1 shows estimated intervals of Weibull distribution parameters of the variables).

Range of values for parameters in Table 1 covers values obtained from an observation of a real tram system sample that consists of different vehicles according to operation period and mark. When evaluating an estimation accuracy, one need to test range of parameter values instead of one combination of these values. It is worth to mention that shape coefficient of time between failure random variable is close to 1. Hence, it is neither distinctly ‘‘infant mortality’’ nor ‘‘aging’’ process.

In the reliability theory, the two approaches have evolved from performing maintenance tasks. First, there are known models in which a system or its components fail and there are no visible symptoms about the forthcoming failure (assuming that it is a sudden failure). The majority of the maintenance modelling concepts, being discussed in the literature since the early 1960s, regards to this kind of a problem investigation. In this area, there can be found literature on analytical models development which investigates maintenance policy parameters, spare parts provision and service station (Werbińska, 2008). However, the number of solutions is scarce. It is connected with the necessity of system structural parameters (like redundancy, maintenance capability) and decision variables (like spare parts level, preventive maintenance strategy parameters, service station organization tasks) integration, what is the mathematically difficult issue to solve (De Smidt-Destombes, van der Heijden, & van Harten, 2004). Thus, most of the known analytical models are based on the simplified assumptions which limited their practical use in real-life technical systems modelling. The fundamental methods used in this area encompass queuing theory implementation (see e.g. (Gross & Pinkus, 1979; Mokhles & Saleh, 1988; Subramanian & Natarajan, 1982)) or simulation methods use (see e.g. (De Smidt-Destombes et al., 2004, De Smidt-Destombes, van der Heijden, & van Harten, 2006, 2007)).

Table 2 summarises the state of art of queuing models of technical systems with repair facilities based on Kendal’s notation. Meaning of symbols in system description in Table 2 is different than in *A*|*E*|*D* tram system model notation.

The second approach involves modelling problems of systems/its components, when assuming that failure is not a sudden one. In this approach, before a component breaks down, there will be some sings of reduced performance or abnormalities. The time between the first identification of abnormalities (called initial point) and the actual failure time (failure point) will vary depending on the deterioration rate of the component. This time period is called a delay time to carry out maintenance or an inspection (Pillay, Wang, & Wall, 2001), and the modelling concept is called a delay-time modelling approach. This concept, which provides useful means of modelling the effect of periodic inspections on the failure rate of repairable technical systems, was developed by Christer et al., see e.g. (Christer, 1987, 1982; Christer & Waller, 1984a, 1984b; Christer & Whitelaw, 1983).

The described above analytic approaches are not sufficient to find relationships: hazard probability of the tram system and mean hazard time as functions of number of spare trams when at least one random variable from the *A*, *E*, and *D* is not exponential.

In the paper, exact solutions and estimations of the hazard probability of the tram system and mean hazard time as functions of number of spare trams are presented. For M/W/M tram system models, where M denotes that time between two subsequent failures and delivery time are described by exponential random variables, while W means Weibull random variable of exchange time, exact solutions have been obtained. The first is hypo-exponential distribution based one, while the second is order statistics based. These solutions are for models with infinite and finite amount of repair facilities. As regards the W/W/W model, where all three distributions are Weibull, accuracies of two estimations have been verified using simulation experiments. The first one was based upon the order statistics solution for the M/W/M model, and the other one, which is conditional probability distribution based, was dedicated to W/W/W models.

Structure of the paper is as follows. In Section 2, there are general formulae for hazard probability and mean hazard time for tram system with infinite amount of repair facilities. They are common for both exact solutions for M/W/M tram system model (hence, M/M/M model too) and estimations for W/W/M and W/W/W models. In next section, two exact solutions for M/W/M models are given. In Sections 4 and 5 there are estimations for W/W/M and W/W/W models. Then the comparison of our approach and balance equation method is presented. In Section 7, accuracy and computational complexity of exact solutions and estimations are given. Next, generalizations with emphasis put on tram system

Table 1
Parameters of Weibull distribution of tram dependability measures.

Variable	Symbol	Lambda		*k*	
		min	max	min	max
Time between failures	*A*	0.007	0.020	0.90	1.10
Exchange time	*E*	0.020	0.050	1.00	1.50
Repair and delivery time	*D*	0.005	0.010	1.00	1.50