European Journal of Operational Research 231 (2013) 328–336

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

The risk-averse newsvendor problem with random capacity

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ARTICLE INFO

Article history:
Received 16 September 2012
Accepted 28 May 2013
Available online 5 June 2013

Keywords:
Conditional Value-at-Risk
Value-at-Risk
Newsvendor problem
Capacity uncertainty

ABSTRACT

We study the effect of capacity uncertainty on the inventory decisions of a risk-averse newsvendor. We consider two well-known risk criteria, namely Value-at-Risk (VaR) included as a constraint and Conditional Value-at-Risk (CVaR). For the risk-neutral newsvendor, we find that the optimal order quantity is not affected by the capacity uncertainty. However, this result does not hold for the risk-averse newsvendor problem. Specifically, we find that capacity uncertainty decreases the order quantity under the CVaR criterion. Under the VaR constraint, capacity uncertainty leads to an order decrease for low confidence levels, but to an order increase for high confidence levels. This implies that the risk criterion should be carefully selected as it has an important effect on inventory decisions. This is shown for the newsvendor problem, but is also likely to hold for other inventory control problems that future research can address.

1. Introduction

In most of the inventory control literature, demand uncertainty is the only type of uncertainty considered. However, there are many reasons why in real life situations capacity may also be uncertain. These include yield loss (related to quality considerations), unreliable machinery and unplanned maintenance. We refer interest readers to Yano and Lee (1995) and Grosfeld-Nir and Gerchak (2004) for summaries of the effects of random capacity on inventory control.

Random capacity, resulting from the before mentioned uncertainties, has been studied for the well-known newsvendor problem. Ciarallo et al. (1994) show that random capacity does not affect the order quantity of the risk neutral newsvendor, i.e., under the profit maximization objective. More general, as discussed in detail in Section 2.1, the effect of random capacity under the risk-averse criterion on the single period problem is limited. However, Schweitzer and Cachon (2000) provide experimental evidence suggesting that inventory managers are risk averse for high-value products. Thus, the assumption of risk neutrality is not always applicable. In view of this, a number of authors have therefore considered the risk aversion issue through the use of some related objectives, such as utility functions, the mean–variance objective function, VaR (downside risk) and CVaR (coherent measures). We will summarize key results in Section 2.2. It will appear that risk aversion has not been studied on inventory systems with random capacity.

In this paper, we consider a risk-averse newsvendor problem with uncertainty in both demand volume and production capacity. We find that the risk criterion should be carefully selected, as different criteria affect the inventory decision in different ways. More specifically, we find that capacity uncertainty always leads to a lower order quantity under the CVaR criterion, but it may lead to either a lower or a higher order quantity under the VaR constraint. So, different from the risk neutral setting, we find that additional uncertainty may lead to a lower inventory level under both considered risk averse criteria.

The remainder of this paper is organized as follows. In the next section, we review the literature on the newsvendor problem with random capacity and on the risk-averse newsvendor problem. Section 3 introduces the risk-averse newsvendor model and the various criteria. In Section 4, we derive the optimal order decisions under different risk measures. Section 5 discusses the impact of the randomness of capacity and the mean-CVaR criterion. Section 6 concludes the paper.

2. Literature review

The newsvendor literature is surveyed from two aspects: random capacity/yield and risk averseness. We mainly focus on the single-period newsvendor problem, but also consider more general periodic review models that include the newsvendor model as a special case.

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0377-2217/$ - see front matter
http://dx.doi.org/10.1016/j.ejor.2013.05.044

2.1. The newsvendor problem with random capacity/yield

Ciarallo et al. (1994) first study of the implications of variable capacity for the newsvendor problem. They study a periodic-review production model with random capacity, and show for the single-period problem that random capacity does not affect the optimal order quantity. Wang and Gerchak (1996) extend their model to a system with both random capacity and random yield. Bollapragada et al. (2004) consider assembly systems with random capacity at each component supplier that are controlled with base-stock policies. Erdem et al. (2006) consider an EOQ model with multiple suppliers who face random capacities. They obtain the unique optimal order quantities for the suppliers under which the expected unsatisfied order is the same for each supplier, and study the diversification among suppliers for some given distributions. Okyay et al. (2010) consider the classical newsvendor problem with random demand, random capacity and/or random yield where the demand and supply are assumed to be dependent. They provide mathematical characterizations of the optimal order quantity.

This research area related to random yield, started by Karlin (1958), has received a lot of attention in operations research. Basu (1987) characterizes the optimal order policy when the mean of random supply is proportional to the quantity ordered. Hsu and Bassok (1999) study the optimal production strategy for multiple products with downward substitution. Khouja (1999) provides a comprehensive review of research before 1995 in this area. Some renewed interest and new applications have appeared in this area in recent years. Grosfeld-Nir et al. (2000) develop a framework to calculate the optimal batch and the expected number of inspections with uncertain yield patterns. Under an acquisition-price-sensitive yield rate, Bakal and Akcali (2006) analyze the impacts of yield variation on the profitability of remanufacturing and the value of perfect yield rate information. Rekik et al. (2007) analyze the optimal order quantity in the presence of two types of errors caused by yield uncertainty: the standard deviation of the received quantity is independent of the quantity ordered and the standard deviation of the quantity received is proportional to the quantity ordered. All these authors assume a risk neutral newsvendor, different from our research.

2.2. The risk-averse newsvendor problem

Modelling risk-averse newsvendor problems has received considerable attention in recent years. Utility functions (e.g., Lau (1980), Agrawal and Seshadri (2000), Chen et al. (2007), Choi and Ruszczyński (2011)), mean–variance (e.g., Chen and Federgruen (2000), Wu et al. (2009)) and VaR/CVaR are the three main research streams. We refer interested readers to Qin et al. (2011) for summaries of the ordering policies of newsvendors with various risk preferences. Here, we review the key contribution of the VaR/CVaR approach, which are the two most important financial risk measures that have emerged and been widely used in recent years.

Luciano et al. (2003) use the VaR as a risk measure in a single product, multi-period inventory model. They give an exact analysis to obtain the VaR. Gan et al. (2005) incorporate the VaR concept for a newsvendor problem with a downside risk constraint for a single product. They study channel coordination without shortage cost where the supplier is risk-neutral and the retailer is constrained by a downside risk. Özler et al. (2009) consider the single-period newsvendor problem with VaR constraints in the multi-product case. They derive the exact profit distribution function for the two-product newsvendor problem and develop an approximation method for the profit distribution of multi-product case. Jammernegg and Kischka (2012) present a comparative analysis of VaR and CVaR as objectives and constraints, respectively, in the newsvendor problem. They show that the CVaR newsvendor orders less than the VaR newsvendor.

Gotoh and Takano (2007) investigate the risk-averse newsvendor problem under the objective of minimizing the CVaR. They show that downside risk measures including the CVaR are tractable due to their convexity. Ahmed et al. (2007) study the multi-period newsvendor problem without shortage cost, where the objective function is a coherent risk measure. They show that the structure of the optimal solution of the risk-averse model is similar to that of the risk-neutral case. Jammernegg and Kischka (2007) analyze the ordering policy and its corresponding performance measures, such as the cycle service level. They consider the trade-off between the CVaR and the expected profit and conclude that a risk-averse (risk-seeking) newsvendor orders less (more) than a risk-neutral newsvendor. Chen et al. (2009) analyze a price-dependent newsvendor model under the CVaR criterion, and derive the optimal ordering and pricing decisions for both the additive and multiplicative demand model. Choi et al. (2011) consider a multi-product risk-averse newsvendor without shortage cost under the law-invariant coherent measures of risk and show that increased risk aversion leads to decreased orders. Xu and Li (2010) discuss the newsvendor problem with shortage cost by balancing the expected profit and the CVaR criterion. It is shown that the optimal order quantity of the risk-averse newsvendor can be either larger or smaller than that of the risk-neutral newsvendor.

A common assumption of all the literature on VaR and/or CVaR approach is, to the best of our knowledge, that the only source of uncertainty is the demand. An interesting and unexplored question is what happens in an, arguably even more realistic, situation with random capacity. This is the main focus of our research, where we will consider both the VaR and the CVaR approach in order to gain insights into the effects of different risk criteria.

3. The newsvendor problem with multiple criteria

In this section, we define the newsvendor problem under three different objectives. The notations used in the paper are given in Table 1. Random demand and random capacity are assumed to be independent. Without loss of generality, we assume that $F(0) = G(0) = 0$.

If Q units are ordered, the retailer's profit for unlimited capacity is

$$\pi_c(Q) = (r-c)Q - (r-s)(Q-D)^+, \tag{1}$$

the profit for random capacity is

$$\pi_s(Q) = (r-c)\min\{Q,A\} - (r-s)(\min\{Q,A\} - D)^+, \tag{2}$$

and so the expected profit $\pi_s(Q)$ is

$$\begin{aligned} E(\pi_s(Q)) &= (r-c)\int_0^Q y\,dG(y) + (r-c)(1-G(Q))Q \\ &\quad - (r-s)\int_0^Q \int_0^y (y-x)\,dF(x)\,dG(y) \\ &\quad - (r-s)(1-G(Q))\int_0^Q (Q-x)\,dF(x). \end{aligned}$$

The expected profit, VaR and CVaR are three main approaches of modelling newsvendor problem. The expected profit objective as the traditional risk neutral criterion is to maximize the expected profit.

VaR measures the maximum profit loss in the majority of cases (normally 95% or 99%). The standard VaR criterion maximizes the lower end of a one-sided confidence interval (Artzner et al. (1999)). A clear disadvantage of the standard VaR criterion is that it purely considers risk but not the expected profits. Although