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Specification and estimation of multiple output technologies: A primal approach ☆

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ABSTRACT

This paper addresses specification and estimation of multiple-outputs and multiple-inputs production technology in the presence of technical inefficiency. The primary focus is on the primal formulations. Several competing specifications such as production function, input (output) distance function, input requirement function are considered. We show that all these specifications come from the same transformation function and are algebraically identical. We also show that: (i) unless the transformation function is separable (i.e., outputs are separable from inputs), the input (output) ratios in the input (output) distance function can not be treated as exogenous (uncorrelated with technical inefficiency) resulting inconsistent estimates of the input (output) distance function parameters. (ii) Even if input (output) ratios are exogenous, estimation of the input (output) distance function will result in inconsistent parameter estimates if outputs (inputs) are endogenous. We address endogeneity and instrumental variable issues in details in the context of flexible (translog) functional forms. Estimation of several specifications using both single and system approaches are discussed using Norwegian dairy farming data.

1. Introduction

Although most of the production processes involve multiple inputs and multiple outputs, estimation of multiple inputs and multiple outputs production function is not popular especially when a single equation approach is used. The problem is that in estimating a production function (using OLS or nonlinear least squares, NLS) one of the output (usually the dependent variable) is considered endogenous[1] and the rest of them (along with the inputs) are treated as exogenous. If all outputs are endogenous, estimation results using the production function suffer from the endogeneity problem since endogeneity of only one output is recognized, i.e., the one used as the dependent variable. Furthermore, results differ depending on which output is chosen as the dependent variable. If the inputs are also endogenous, the endogeneity problem is magnified because in this case all the regressors will be endogenous. To avoid this problem researchers use distance function formulations while estimating multiple-inputs and multiple-output technologies. However, the distance functions cannot avoid the endogeneity problem completely. For example, in estimating the input distance function (IDF) (Shephard (1953, 1970)) the maintained hypothesis (standard assumption) is that outputs are exogenously given. While this might be true for service and demand determined industries such as banks, airlines, railroads, post offices, and public utilities, outputs for majority of manufacturing firms and agricultural farms are unlikely to be exogenously given (except perhaps the case when there are explicit quotas on outputs). Thus, results from the IDF models might also suffer from endogeneity problem, especially if outputs are endogenous. Similarly, in estimating output distance functions (ODFs) the implicit assumption is that inputs are exogenously given (which is rarely the case in practice). Thus the ODF results will be inconsistent if inputs are not exogenous. Since the outputs (inputs) are assumed to be exogenous (endogenous) in IDF (ODF), one needs to check appropriateness of the endogeneity/exogeneity assumptions either from econometric or from economic considerations. The question then is: if outputs (inputs) are endogenous, can one treat the input (output) ratios as exogenous in the IDF (ODF)? If so one needs instruments for the output (input) variables in the IDF (ODF) models. If not, instruments for all input and output variables are required irrespective of whether an IDF or an ODF is used. Although there are papers on input and output distance functions in which instrumental variables are used, it is not clear

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[1] An input (output) is said to be endogenous if it is correlated with the error term in the estimating equation. Since we model inefficiency in production explicitly via a random variable in addition to the usual random noise, an input (output) can be endogenous if it is uncorrelated with the noise component but correlated with the inefficiency component.

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which regressors are endogenous and in what specifications.[2] That is, the readers do not get a clear idea of whether to worry about endogeneity of input ratios or outputs or both while using an IDF.

In this paper we address the endogeneity issue from the behavioral assumption that producers maximize profit and both inputs and outputs are decision/choice variables. We focus mostly on the distance function formulations because the endogeneity problem is obvious for the production and input requirement functions (in which inputs and outputs appear as regressors) but is not so obvious for distance functions since some of the regressors appear in ratio form.[3] Färe and Primont (1995) dealt with the theoretical issues associated with multi-output distance function models in details. However, they did not discuss estimation issues and therefore the endogeneity issue never arose in their discussion. Kumbhakar and Lovell (2000) discussed the issues in terms of the dual profit function in which endogeneity problem did not arise because input and output prices are assumed to be exogenous. Kumbhakar (2012) discusses the endogeneity issue in the context of single output production technology with no inefficiency. Here we take a similar approach with multiple output technologies with inefficiency. Our primary concern here is estimation and we examine the endogeneity issue in details using flexible form of the transformation function (instead of a dual profit function) with multiple inputs and multiple outputs from which the IDF and ODF are derived. The idea is to examine when and where one can treat the input and output ratios as exogenous. We consider the translog formulation to show that the endogeneity problem stays unless the IDF (ODF) is rewritten where all the regressors are in ratio form.

The rest of the paper is organized as follows. In Section 2 we deal with specification of different formulations using the translog functional form. Estimation issues of these models are discussed in Section 3 from a single equation perspective. Section 4 addresses estimation issues using a system approach. The data used in the paper is discussed in Sections 5 and 6 reports empirical results. Finally, Section 7 summarizes the main results and conclusions of the paper.

2. Representations of the transformation function

Consider a production process in which M outputs are produced using J inputs and the technology is specified as $A\ f(\theta x, \lambda y) = 1$, where x is a vector of K inputs and y is a vector of M outputs. The A term captures the impact of observed and unobserved factors that affect the transformation function neutrally.[4] Input technical inefficiency is indicated by $\theta \leqslant 1$ and output technical inefficiency is captured by $\lambda \geqslant 1$ (both are scalars). Thus, $\theta x \leqslant x$ is the input vector in efficiency (effective) units so that if $\theta = 0.9$ inputs are 90% efficient (i.e., use of each input could be reduced by 10% without reducing outputs, if inefficiency is eliminated). Alternatively, $\theta \leqslant 1$ is input-oriented efficiency. Similarly, if λ is 1.05, each output could be increased by 5% without increasing any input, when inefficiency is eliminated. Thus $\lambda^{-1} \leqslant 1$can be viewed as output-oriented efficiency. Since both θ and λ are not identified, we consider the following special cases. If $\theta = 1$ and $\lambda \geqslant 1$ then we have output-oriented technical inefficiency. Similarly, if $\lambda = 1$ and $\theta \leqslant 1$ then we have input-oriented technical inefficiency. Finally, if $\lambda \cdot \theta = 1$ technical inefficiency is said to be hyperbolic. It says that if the inputs are contracted by a constant proportion, outputs are expanded by the same proportion. That is, instead of moving to the frontier by either expanding outputs (keeping the inputs unchanged) or contracting inputs (holding outputs unchanged), the hyperbolic measure chooses a path to the frontier that leads to a simultaneous increase in outputs and a decrease in inputs by the same rate.

There are several other ways of modeling the primal technology. In a directional input distance function (Briec, 1997, Chambers et al., 1998 a path to the frontier can be chosen in a way so that the inputs and outputs decrease (increase) at different rates. In a gauge function (McFadden, 1978; Han, 2012) inputs and outputs increase equi-proportionally to reach the frontier.[5] In this paper we specify the technology in terms of the transformation function $f(\cdot)$ because it is much more general than the production/distance/input requirement function. Furthermore, we bring economic behavior into the analysis to address endogeneity of inputs and outputs.

2.1. The translog transformation function

We rewrite the transformation function as $Af(y^*, x^*) = 1$, where $y^* = y\lambda$, $x^* = x\theta$, and $f(y^*, x^*)$ is assumed to be translog (TL), i.e.,

TL transformation function : $\ln f(y^*, x^*)$
$$= \sum_m \alpha_m \ln y_m^* + \frac{1}{2}\sum_m\sum_n \alpha_{mn} \times \ln y_m^* \ln y_n^* + \sum_j \beta_j \ln x_j^* + \frac{1}{2}\sum_j\sum_k \beta_{jk} \ln x_j^* \ln x_k^* + \sum_m\sum_j \delta_{mj} \ln y_m^* \ln x_j^*, \qquad (1)$$

The above function is assumed to satisfy the following symmetry restrictions, viz., $\beta_{jk} = \beta_{kj}$ and $\alpha_{mn} = \alpha_{nm}$. One can use the following normalizations ($\alpha_1 = -1,\ \alpha_{1n} = 0,\ \forall n,\ \delta_{1j} = 0,\ \forall j,\ \theta = 1$) to obtain a pseudo production function, viz.,

TL Production function :
$$\ln y_1 = \ln A + \sum_j \beta_j \ln x_j + \frac{1}{2}\sum_j\sum_k \beta_{jk} \ln x_j \ln x_k + \sum_{m=2} \alpha_m \ln y_m + \frac{1}{2}\sum_{m=2}\sum_{n=2} \alpha_{mn} \ln y_m \ln y_n + \sum_{m=2}\sum_j \delta_{mj} \ln y_m \ln x_j + u \qquad (2)$$

where

$$u = \ln\lambda\left(-1 + \sum_{m=2}\alpha_m + \sum_{m=2}\sum_{n=2}\alpha_{mn}\ln y_n + \sum_{m=2}\sum_j \delta_{mj}\ln x_j\right) + \frac{1}{2}\sum_{m=2}\sum_{n=2}\alpha_{mn}(\ln\lambda)^2.$$

[2] Kumbhakar (2012) is an exception, although this paper assumes that producers are fully efficient and produce a single output.

[3] Kumbhakar (1996) addressed the endogeneity issue in a multiple production model with technical and allocative inefficiency in terms of a profit function. The main problems in dealing with translog profit function are the following. First, it cannot handle negative profit which is quite common in reality; second, estimation of profit function relies exclusively on input and output prices which are often difficult to get and variations in prices are often very little (which makes the parameter estimates imprecise). Because of these problems we formulate the problem in a primal framework so that information on input and output quantities can be directly used.

[4] It can be viewed as TFP growth under unitary returns to scale.

[5] Cherchye et al., 2010 discuss use of gauge function in a profit maximizing model. Although the directional distance function talk of choice of directions endogenously, to our knowledge no one has estimated such a system econometrically. Our system approach addresses the endogenous choice using a transformation function. Thus, it can be linked with the directional distance function literature. However, since no behavioral assumption is made in directional distance function models, when it comes to empirical application the choice of direction is arbitrarily decided leaving a big hole between the elegant theory and the inelegant econometric model.