European Journal of Operational Research 231 (2013) 503–506

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Short Communication

Endogenous production capacity investment in natural gas market equilibrium models

Daniel Huppmann *

German Institute for Economic Research (DIW Berlin), Mohrenstraße 58, 10117 Berlin, Germany

ARTICLE INFO

Article history:
Received 24 January 2013
Accepted 30 May 2013
Available online 6 June 2013

Keywords:
Natural gas
Equilibrium model
Endogenous investment
Capacity expansion
Logarithmic cost function

ABSTRACT

The large-scale natural gas equilibrium model applied in Egging, 2013 combines long-term market equilibria and investments in infrastructure while accounting for market power by certain suppliers. Such models are widely used to simulate market outcomes given different scenarios of demand and supply development, environmental regulations and investment options in natural gas and other resource markets.

However, no model has so far combined the logarithmic production cost function commonly used in natural gas models with endogenous investment decisions in production capacity. Given the importance of capacity constraints in the determination of the natural gas supply, this is a serious shortcoming of the current literature. This short note provides a proof that combining endogenous investment decisions and a logarithmic cost function yields a convex minimization problem, paving the way for an important extension of current state-of-the-art equilibrium models.

1. Introduction

The natural gas model applied in Egging (2013) is one of the latest in a series of equilibrium models for this particular fuel. The interest stems from several potentially game-changing trends: liberalization of natural gas markets, carbon dioxide emission constraints and an expected replacement of coal by relatively clean natural gas in Europe (EC Energy Roadmap, 2011); unconventional reserves in North America and other regions (IEA, 2011); and frequent concerns regarding supply security and European dependence on a small number of suppliers (cf. Leveque et al., 2010).

Hence, a number of equilibrium models have been developed over the past decade to provide numerical analysis of different scenarios regarding future supply and demand patterns, environmental regulation and infrastructure investment options. Two large-scale natural gas equilibrium models are the *GASTALE* model, developed by ECN (Lise & Hobbs, 2008), and the *World Gas Model (WGM)*, joint work by the University of Maryland and DIW Berlin (Egging, Holz, & Gabriel, 2010).[1] These models share a number of characteristics, similar to the model in Egging (2013): they are spatial partial equilibrium models with a detailed geographic disaggregation, allowing for analysis and comparison of different pipeline and LNG export/import options; they consider seasonality within a year and explicitly model storage to shift natural gas between low- and high-demand seasons; they are multi-period models and endogenously determine optimal investment in infrastructure[2]; and they allow for oligopolistic behavior by (a subset of) suppliers, i.e., Cournot competition. Furthermore, all these models apply a logarithmic cost function, as first proposed by Golombek, Gjelsvik, and Rosendahl (1995), in order to capture the specific characteristics of natural gas production: sharply increasing costs when producing close to full capacity.

However, none of these models allows for endogenous investment in production capacity; instead, the production capacity in future periods is defined exogenously. Given that production capacity is a significant determinant of results and that these models simulate price and quantity trajectories for several decades into the future, this omission is certainly a major drawback. It is owed, in all likelihood, to the rather complicated functional form when including investment decision variables in the logarithmic cost function. This paper provides the proof that this extension yields a convex problem, which is a prerequisite for solving this problem as an equilibrium model.

Let me also mention one other recent natural gas model: the *GaMMES* model was developed by EDF and IFPEN (Abada, Gabriel, Briat, & Massol, 2013). In contrast to the models presented above, it distinguishes between spot market sales and long-term contracts. It also assumes a slightly different formulation of the logarithmic

* Tel.: +49 30 897 89 378.
E-mail address: dhuppmann@diw.de

[1] Both models were published in different versions and used extensively for scenario simulations; only one recent publication for each model is cited here.

[2] GASTALE implements a rolling-horizon investment model, while WGM is an open-loop equilibrium model.

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http://dx.doi.org/10.1016/j.ejor.2013.05.048

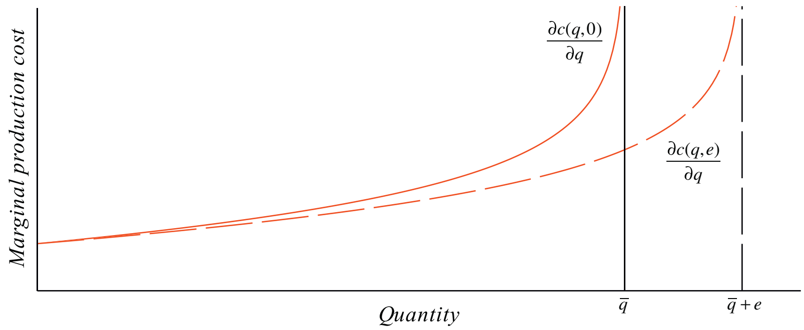

Fig. 1. Illustration of the marginal cost function (production capacity $\bar{q}$) without investment (—) and with additional investment e (- -).

cost function: production costs are not increasing relative to capacity utilization, as in the other models, but relative to remaining reserves. This is an interesting approach, but differs from what is discussed here.

All these models are formulated as Mixed Complementarity Problems (MCP). The optimization problems of different players subject to engineering and other constraints are solved simultaneously by deriving their respective Karush–Kuhn–Tucker (KKT) conditions, combined with market clearing constraints. The MCP framework is convenient for this type of exercise, as it allows to include Cournot market power for certain suppliers, in contrast to welfare maximization or cost minimization problems. In addition, these models can easily be extended to include stochasticity (e.g., Gabriel, Zhuang, & Egging, 2009) or two-level problems such as Stackelberg competition (e.g., Siddiqui & Gabriel, 2013).

2. Mathematical formulation

Assume a supplier with decision variables q_y (production quantity) and e_y (production capacity expansion/investment). The periods are denoted by $y \in \{1, \ldots, \bar{y}\}$. In order to keep the notation concise, y denotes both a period as well as its position in the set. Hence, $\bar{y}$ stands for both the last period as well as the number of periods in the set. Following this logic, I use $y' < y$ for "all periods y' prior to period y" in sums and indices, and $y' > y$ for the inverse statement. The price at which the produced quantity is sold is denoted by p_y, and the initial production capacity is $\bar{q}$.

Production costs $c_y(\cdot)$ are determined by a logarithmic cost function as introduced by Golombek et al. (1995) related to capacity utilization (see Eq. (3a) below).[3] This function is illustrated in Fig. 1: marginal production costs increase sharply when operating close to capacity. Hence, if capacity is expanded, marginal production costs for the same quantity decrease.

In line with the literature cited above, capacity investment costs are assumed to be linear. The parameters of the cost function are denoted by greek letters and may vary by period: α_y, β_y, γ_y are the parameters for the production cost function; κ_y is the (linear) unit production capacity investment cost. All cost parameters are non-negative. Discounting of future profits may be implicitly included in the price and cost parameters. For now, I abstract from Cournot market power and other considerations such as reserve horizon or maximum investment constraints. These extensions are briefly discussed below.

Furthermore, I assume that γ_y is strictly positive. As a consequence, the production quantity q_y is implicitly bounded from above; this is explained in more detail below. I can therefore omit an explicit production capacity constraint. Otherwise, the problem would reduce to a quadratic optimization problem under linear capacity constraints, and hence convexity would hold trivially.

The profit maximization problem of the supplier can then be written as follows, converted to a minimization problem:

$$\min_{q,e} f(q,e) = \sum_y -p_y q_y + c_y(q_y, e_1, \ldots, e_{y-1}) + \kappa_y e_y \tag{1}$$
$$\text{s.t. } q, e \in \mathbb{R}_+^{\bar{y}}$$

This yields the following Karush–Kuhn–Tucker (KKT) conditions:

$$-p_y + \frac{\partial c_y(\cdot)}{\partial q_y} \geqslant 0 \perp q_y \geqslant 0 \tag{2a}$$

$$\sum_{y'>y} \frac{\partial c_{y'}(\cdot)}{\partial e_y} + \kappa_y \geqslant 0 \perp e_y \geqslant 0 \tag{2b}$$

This optimization problem relates to Eq. (3.1.1) in Egging (2013). The KKT condition concerning the production quantity relates to Eq. (3.7.16).

It is straightforward to see that there will never be investment in the last period; the KKT condition reduces to $0 + \kappa_y \geqslant 0$, implying $e_y = 0$ if $\kappa_y > 0$. This variable and the associated equation can thus be omitted from further consideration.

The production cost function and its partial derivatives are listed below. In order to make the notation more concise, the sum of previous investments, $\sum_{y'<y} e_{y'}$, is replaced by $e(y)$ for the remainder of this work.

$$c_y(\cdot) = (\alpha_y + \gamma_y) q_y + \beta q_y^2 + \gamma_y (\bar{q} + e(y) - q_y) \ln\left(1 - \frac{q_y}{\bar{q} + e(y)}\right) \tag{3a}$$

$$\frac{\partial c_y(\cdot)}{\partial q_y} = \alpha_y + 2\beta_y q_y - \gamma_y \ln\left(1 - \frac{q_y}{\bar{q} + e(y)}\right) \tag{3b}$$

$$\frac{\partial c_y(\cdot)}{\partial e_{\hat{y}}} = \gamma_y \ln\left(1 - \frac{q_y}{\bar{q} + e(y)}\right) + \gamma_y \frac{q_y}{\bar{q} + e(y)} \quad \text{if } \hat{y} < y \tag{3c}$$

$$\frac{\partial^2 c_y(\cdot)}{\partial q_y^2} = 2\beta_y + \gamma_y \frac{1}{\bar{q} + e(y) - q_y} \tag{3d}$$

$$\frac{\partial^2 c_y(\cdot)}{\partial e_{\hat{y}} \partial e_{\tilde{y}}} = \gamma_y \frac{q_y^2}{(\bar{q} + e(y) - q_y)(\bar{q} + e(y))^2} \quad \text{if } \hat{y} < y \wedge \tilde{y} < y \tag{3e}$$

$$\frac{\partial^2 c_y(\cdot)}{\partial q_y \partial e_{\hat{y}}} = -\gamma_y \frac{q_y}{(\bar{q} + e(y) - q_y)(\bar{q} + e(y))} \quad \text{if } \hat{y} < y \tag{3f}$$

Given this cost function and assuming $\gamma_y > 0\ \forall y$, marginal production costs tend to infinity when the produced quantity tends to initial capacity plus expansions in previous periods. Hence, production quantity q_y is implicitly bounded by capacity. Mathematically speaking, for any $p_y > 0$, there exists a quantity q_y with $\frac{\partial c_y(\cdot)}{\partial q_y} \geqslant p_y$ and $q_y < \bar{q} + e(y)$. Hence, an explicit production capacity condition is not required as a constraint in the optimization problem (1).[4]

[3] The production cost function is not explicitly specified in Egging (2013), but it is discussed in detail in the dissertation referenced in the article.

[4] Since Egging (2013) does not assume $\gamma_y > 0$ for all suppliers and years, the capacity constraint is included there explicitly (Eqs. (3.1.2) and (3.7.17)).