European Journal of Operational Research 227 (2013) 464–470

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Inventory allocation and shipping when demand temporarily exceeds production capacity

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ARTICLE INFO

Article history:
Received 14 February 2012
Accepted 9 January 2013
Available online 17 January 2013

Keywords:
Inventory allocation
Shipping
Customer waiting cost
Supply chain management

ABSTRACT

We address the concept of an integrated inventory allocation and shipping model for a manufacturer with limited production capacity and multiple types of retailers with different backorder/waiting and delivery costs. The problem is to decide how to allocate and deliver produced items when the total retailer demand exceeds the production capacity, so that total retailer backorder and delivery costs are minimized. Our analytical model provides optimal allocation and shipping policies from the manufacturer's viewpoint. We also investigate the allocation strategy of a manufacturer competing with other retailers to directly sell to end consumers.

1. Introduction

In this paper, we consider a manufacturer who has a limited production capacity and sells a single item to multiple retailers. The orders of retailers are not known in advance, so the manufacturer produces a certain number of items, possibly up to his maximum production capacity, for the next period. At the beginning of the next period, retailer orders materialize. When the sum of retailer orders is larger than the available inventory, an allocation problem occurs. Because the stock and the production capacity are limited resources, products must be allocated among retailers according to their relative importance and associated costs. The allocated items are shipped immediately. Any unsatisfied demand is delivered as the manufacturer produces additional items. The manufacturer pays retailer specific backorder/waiting costs, according to the delivery time and quantity of the backordered items. The fixed cost per additional shipment is also considered.

There are two fundamental decisions in this case. The first decision concerns allocating inventory, which can be regarded as a critical asset in the system. Different retailers may have different values for the manufacturer, so it is important to satisfy some retailers over others. The second decision is related to the delivery frequency and sequence because retailers may have different delivery costs. Thus, the optimal inventory policy should depend on both delivery issues and the importance of retailers.

It is common to see higher demand than production capacity, especially when a new product is introduced to the market. Because the high demand is temporary and decreases below the production capacity fairly soon, the manufacturer does not want to invest his time and money in increasing the production capacity. In this situation, the manufacturer needs to efficiently allocate their inventory and deliver new production to retailers in several shipments, partially fulfilling the demand.

This has happened several times in practice. Popular Apple products from the iMac to the iPad have often experienced higher demand than supply for several months following a product launch (Wilcox, 2002; Heine, 2011). Nintendo's market-dominating Wii console had more demand than production capacity for over 2 years, from the product launch in November 2006 to early 2009, even though it produced 1.8 million units a month in the beginning and 2.4 million units a month after a capacity expansion in 2008 (Reisinger, 2009). Similar situations can happen when the supply is disrupted temporarily, such as after the earthquake and tsunami in Japan in 2011 (Hookway and Watcharasakwet, 2011).

The allocation and shipping problem becomes particularly interesting when multiple sales channels are considered. According to Wilcox (2002) and Wingfield (2004), independent retailers accused Apple of not allocating enough of their popular iMac computers and iPod music players inventory, which were readily available through Apple stores. Geng and Mallik (2007) also report that a new Sony electronic product typically showed up at the sonystyle.com website several months before the same product was available in the retail channel.

We next review related articles and identify key differences between other models and ours. The basic inventory allocation problem has been studied in various contexts. However, many studies use simple assumptions in their models, such as uncapacitated

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0377-2217/$ - see front matter
http://dx.doi.org/10.1016/j.ejor.2013.01.013

production, constant lead times, and/or identical customers. Jackson (1988) and McGavin et al. (1993) discuss basic optimal stock allocation problems in periodic review systems. Cachon and Lariviere (1999a) consider a single supplier with limited capacity selling to several retailers. They evaluate several allocation mechanisms for truth-eliciting properties. A generalization of the model is given in Deshpande and Schwartz (2005). Cachon and Lariviere (1999b) consider a capacity allocation problem using the turn-and-earn allocation scheme. Cachon and Lariviere (1999c) develop an equilibrium analysis of linear, proportional, and uniform allocation of scarce capacity. Dai et al. (2006) study capacity allocation and inventory policy in a distribution system. They find optimal allocation or channel coordination mechanisms for centralized and decentralized systems, respectively. While production and allocation issues are well-studied in these papers, the delivery or shipping process is not investigated, because there is no fixed setup or transportation cost. On the other hand, we investigate the integration of inventory allocation and delivery policies considering customer waiting and fixed delivery costs.

While there are many articles that address the integration issues in production and distribution systems, many of these models only focus on transportation issues such as vehicle routing and shipment size. Most of them do not consider inventory allocation and customer waiting issues. Erenguc et al. (1999), Sarmiento and Nagi (1999), Bilgen and Ozkarahan (2004), and Chen (2000, 2010) present good reviews on the topic. Blumenfeld et al. (1991) study a model with one manufacturer and multiple customers, where the manufacturer produces multiple products, one for each customer. Each product is allowed to be produced multiple times within a production cycle, and the authors derive optimal production and delivery cycle lengths. Chen and Vairaktarakis (2005) and Pundoor and Chen (2005) show that there is a significant benefit by using the optimal integrated production–distribution schedule compared to the schedule generated by a sequential approach. Chen and Pundoor (2006) and Li and Vairaktarakis (2007) consider the trade-off between the total transportation cost and the customer service performance measured in integrated models. Jang (2006) and Jang and Kim (2007) consider integrated production, allocation, and distribution problems similar to ours, yet with a focus on the optimal decision of production quantity under one direct shipping assumption.

Many authors have shown the economic advantages of using an integrated decision model over a decoupled approach. However, the research on integrated production and distribution models at the detailed level is fairly recent. Our model attempts to optimize detailed order decisions including inventory allocation and delivery policy.

Our work is also somewhat related to the channel conflict problem although we do not directly focus on the issue. Tsay and Agrawal (2004) and Agatz et al. (2007) provide extensive reviews of this literature. Boyaci (2005) considers a multi-channel supply chain in presence of both vertical and horizontal competition. However, Boyaci's work, like most of the other works cited on channel conflict (such as Aydin and Christopher, 2005; Cattani et al., 2006), assumes infinite production capacity. Geng and Mallik (2007) consider a problem similar to ours and establish a Stackelberg game to model inventory competition and allocation in a multichannel supply chain. In their model, both capacitated and infinite games are considered, optimal decisions are sought, and implications of inventory competition and capacity constraints are explored. Research on dual-channel distribution in the setting where the upstream echelon is both a supplier to and a competitor of the downstream echelon has emerged only recently. A number of papers in this stream of literature (e.g., Chiang, 2010; Kumar and Ruan, 2006) focus on channel competition and coordination issues. While these models consider more complicated situations focusing on the channel conflict, they do not consider additional production and delivery issues discussed in this paper.

Although a large number of articles study inventory and capacity allocation models, they typically do not investigate the coordination of inventory allocation and shipping, as studied in this paper. In our scenario, the inventory allocation strategy is partially determined by the delivery sequence, and hence, the manufacturer should not make these decisions in isolation from each other. Therefore, the objective of this study is to compute: (1) inventory allocation at the beginning of the period, and (2) the delivery frequency, size, and sequence of additional production during the period from the point-of-view of a manufacturer to minimize his total cost. Because total demand is greater than production capacity, the manufacturer can make a decision that benefits himself most, instead of the entire supply chain. We believe this is a more practical approach to apply to situations described in this paper. We also evaluate the effect of allocation on channel competition by assuming one of the retailers is a manufacturer.

The remainder of the paper is organized as follows. Section 2 presents mathematical models and a solution approach to determine optimal coordinated decisions. In Section 3, the closed form solutions for special cases and numerical examples are provided. Concluding remarks are given in Section 4.

2. Model development and analysis

In this section, we present a mathematical model and its analysis. The manufacturer needs to make inventory allocation and shipment decisions. In particular, the number, size, and sequence of shipments should be determined. The inventory allocation policy is dependent upon these shipment decisions. We use the following notation in this paper:

p	number of retailers
Q	original inventory level of a manufacturer
D_i	demand from retailer i
Q_i	quantity originally allocated to retailer i
y_i	quantity ordered that is not immediately shipped to retailer i
w_i	backorder (waiting) cost per unit time and item for retailer i
c_i	fixed setup cost per additional shipment to retailer i
n_i	number of additional shipments to retailer i
f	production rate for backordered items

We assume all parameters are non-negative. Q_i, y_i, and n_i are decision variables, and the others are known constants. Throughout the paper we assume that $w_i > w_j$, if $i < j$, without loss of any generality.

Suppose that a manufacturer with Q items in stock collects demand information of p retailers at time $t = 0$. Note that Q can be the production capacity of a manufacturer. The demand D_i can be either obtained from a retailer or estimated by the manufacturer based on historical records and market information. We only consider a non-trivial case such that $\sum_{i=1}^{p} D_i > Q$. The manufacturer allocates Q items to retailers and ships them immediately. Observe that $\sum_{i=1}^{p} Q_i = Q$ and $y_i = D_i - Q_i \geqslant 0$, $i = 1, 2, \ldots, p$.

The backordered items are produced as a rate of f items per unit time. In other words, production time per item is assumed to be $1/f$ of unit time. If y_1 backordered items are delivered to retailer 1 at time t, the manufacturer pays the associated cost for retailer 1 given as $c_1 + w_1 y_1 t$. The value of t depends on the wait time caused by orders from other retailers unless retailer 1 is the one served first. The unit backorder cost may include a loss-of-goodwill penalty as