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Decision Support

Dynamic data envelopment analysis: A relational analysis

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ABSTRACT

This paper proposes a dynamic data envelopment analysis (DEA) model to measure the system and period efficiencies at the same time for multi-period systems, where quasi-fixed inputs or intermediate products are the source of inter-temporal dependence between consecutive periods. A mathematical relationship is derived in which the complement of the system efficiency is a linear combination of those of the period efficiencies. The proposed model is also more discriminative than the existing ones in identifying the systems with better performance. Taiwanese forests, where the forest stock plays the role of quasi-fixed input, are used to illustrate this approach. The results show that the method for calculating the system efficiency in the literature produces over-estimated scores when the dynamic nature is ignored. This makes it necessary to conduct a dynamic analysis whenever data is available.

1. Introduction

Data envelopment analysis (DEA) is a technique for measuring the relative efficiency of a set of decision making units (DMUs) which apply multiple inputs to produce multiple outputs in a period of time. Since the pioneering work of Charnes et al. (1978), numerous studies discussing the methodology and application of DEA have been published; see, for example Cook and Seiford (2009). Not only non-profit organizations, but also for-profit companies find this technique effective in identifying inefficient DMUs, as well as the factors which cause inefficiency.

The conventional DEA technique is devised to measure the performance of a DMU in a specified period of time in a static manner. When several periods with inter-relations are involved, the overall efficiency must be measured in a dynamic manner, taking into account the inter-relationship between consecutive periods. Otherwise, the resulting efficiency measures will be misleading, and there are many papers that investigate inter-temporal effects in performance measurement.

The term "dynamic DEA" means using DEA models to describe the inter-relationships between individual periods and using the associated solution methods to calculate the relative efficiencies for a set of multi-period DMUs. These inter-relationships have different forms. One is that the total amount of an input consumed in all periods must be constant. For example, Färe (1986) measured output efficiency by allowing for input allocation over finitely many periods. In most cases, capital inputs and adjustment costs are two major causes of the dynamic situation, and many papers examine these. For example, Sengupta (1994) used an adjustment cost approach to analyze the influence of risk aversion and output fluctuations on the dynamic production frontier when both current and capital inputs are used to produce outputs.

However, these early works only have one output, and Färe and Grosskopf (1996) introduced the dynamic aspects of production into the conventional DEA model when multi-outputs are involved. They formulated several inter-temporal models, which became the basis for many later studies on dynamic DEA. Sengupta (1999) extended the idea of Sengupta (1994) to incorporate the uncertainty of future input prices. Jaenicke (2000) studied the role of soil capital in a cropping cycle by treating it as an intermediate output. Nemoto and Goto (1999, 2003) distinguished the inputs as variable inputs and quasi-fixed inputs in measuring productivity efficiencies. Their model was modified by Ouellette and Yan (2008) to allow for weaker restrictions on capital investment, and by Von Geymueller (2009) to be able to dispense with price information. Emrouznejad and Thanassoulis (2005) extended the definition of Pareto efficiency to assessment paths to measure efficiencies when inter-temporally dependent input–output levels are caused by capital stock. De Mateo et al. (2006) proposed a range of models to incorporate information on costs of adjustment into the DEA framework. These models are able to identify optimal paths of adjustment for the input quantities, such that the net present value of profit is maximized. Silva and Stefanou (2007) derived bounds on efficiency measures in the context of an adjustment-cost technology and inter-temporal cost minimization. Chen and van Dalen (2010) constructed a model which takes lagged productive effects into account in measuring efficiency. Tone and Tsutsui (2010) developed a slacks-based model to measure the overall and period efficiencies when two consecutive periods are connected by carry-overs.

Most of the above-mentioned studies can only calculate the overall efficiency, and the period-specific efficiencies must be calculated

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separately. Besides, the relationship between the overall and period efficiencies, which is intuitively expected to exist, is not known. Only the additive model of Tone and Tsutsui (2010) is able to measure the overall and period efficiencies at the same time, and it shows that the overall efficiency is an arithmetic average of the period efficiencies from the input side and a harmonic average from the output side. This measure is non-radial, and it is not known whether there exists any such relationship in the conventional radial measure. The purpose of this paper is thus to develop a relational model to calculate the radial measures of the overall and period efficiencies for a multi-period production system, where two consecutive periods are connected by flows. The flow can be capital stock, intermediate output, or any kind of carry-over, e.g. inventory. A mathematical relationship in which the complement of the overall efficiency is a linear combination of those of the period efficiencies will be derived. The model is developed from the multiplier form of the conventional DEA model, and is thus easy to understand.

After the reorganization of forest districts in Taiwan in 1989, the Taiwan Forestry Bureau kept a complete set of data for 3 years in order to measure the effects of this. A characteristic of forest production is that the forest stock left after harvesting in one period becomes the source for growth in the next one. In other words, it is a quasi-fixed input. This set of data is used to investigate the inter-temporal effect of forest stock on the overall efficiency with the model developed in this study.

In the following sections, the relational model is first developed to evaluate the efficiency of a dynamic system, in which two consecutive periods are linked by any kind of flows. The relationship between the overall and period efficiencies calculated from the relational model is then derived, and the overall efficiency is compared with those calculated from other models. Following that, the case of Taiwanese forests is used to illustrate the idea discussed in the preceding sections. Finally, some conclusions are presented.

2. Relational model

In measuring the relative efficiency of a set of n DMUs which use m inputs to produce s outputs over a length of p periods of time, the total quantities over all p periods are generally used. Let $X_{ij}^{(t)}$ and $Y_{rj}^{(t)}$ denote the ith input and rth output, respectively, of the jth DMU in period t. Furthermore, denote $X_{ij} = \sum_{t=1}^{p} X_{ij}^{(t)}$ and $Y_{rj} = \sum_{t=1}^{p} Y_{rj}^{(t)}$ as the total quantities of the ith input and the rth output, respectively, over all p periods. The output-oriented model for measuring the efficiency of DMU k, E_k, under the assumption of constant returns-to-scale by considering all p periods as a whole static system, can be formulated as follows (Charnes et al., 1978):

$$
\begin{aligned}
1/E_k = \min.\quad & \sum_{i=1}^{m} v_i X_{ik} \\
\text{s.t.}\quad & \sum_{r=1}^{s} u_r Y_{rk} = 1 \\
& \sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r Y_{rj} \geqslant 0, \quad j = 1,\ldots,n \\
& u_r, v_i \geqslant \varepsilon, \quad r = 1,\ldots,s, \quad i = 1,\ldots,m
\end{aligned} \tag{1}
$$

where u_r and v_i are virtual multipliers, and ε is a small non-Archimedean number imposed to avoid ignoring any factor in calculating efficiency (Charnes and Cooper, 1984).

The dynamic system considered in this paper is a sequence of periods linked by flows $Z_{fj}^{(t)}$ as depicted in Fig. 1. The concept of flow used in this paper is very generic. It can be a quasi-fixed input where a portion of the output produced in the preceding period is reserved for the production of the current one. An example of this is electricity generation (Nemoto and Goto, 2003). It can also be non-discretionary intermediate products which are completely used for production in the next period, and the role that soil capital plays in generating the rotation effect of crop production (Jaenicke, 2000) is an example of this. No matter whether all or a portion of the flows of a period is used as an input for the next one, the structure of the dynamic system is the same. The portion used for production in the next period is represented by $Z_{fj}^{(t)}$, and the portion as an output of the current period is represented by $Y_{rj}^{(t)}$.

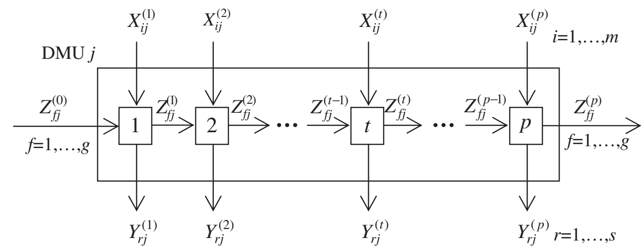

Fig. 1. Dynamic system with flows connecting two consecutive periods.

In studying the performance of a network system composed of several processes connected in series, Kao (2009) developed a relational model to measure the system and process efficiencies at the same time. There are two major characteristics of this model. One is that each process has a constraint associated with it, requiring the aggregate output to be less than or equal to the aggregate input, in addition to the conventional constraints associated with the system. The other is that the same factor has the same multiplier associated with it, regardless of whether it is an input or output in any period. The rationale for this is that the same factor should be valued the same. Based on Kao (2009), Kao and Hwang (2010) formulated the following model to measure the system and process efficiencies for a general series system with p periods:

$$
\begin{aligned}
1/E_k = \min\quad & \sum_{i=1}^{m} v_i X_{ik} \\
\text{s.t.}\quad & \sum_{r=1}^{s} u_r Y_{rk} = 1 \\
& \sum_{i=1}^{m} v_i X_{ij} - \sum_{r=1}^{s} u_r Y_{rj} \geqslant 0, \quad j = 1,\ldots,n \\
& \left(\sum_{i=1}^{m} v_i X_{ij}^{(t)} + \sum_{f=1}^{g} w_f Z_{fj}^{(t-1)}\right) - \left(\sum_{r=1}^{s} u_r Y_{rj}^{(t)} + \sum_{f=1}^{g} w_f Z_{fj}^{(t)}\right) \geqslant 0, \\
& \quad j = 1,\ldots,n; \quad t = 1,\ldots,p \\
& u_r, v_i, w_f \geqslant \varepsilon, \quad r = 1,\ldots,s; \quad i = 1,\ldots,m; \quad f = 1,\ldots,g
\end{aligned} \tag{2}
$$

In addition to the conventional constraints corresponding to the system in Model (1), a set of p constraints corresponding to the operation of the p processes for each DMU, namely, the third constraint set, is added. However, the intermediate products of the first and last periods have been ignored.

The dynamic system discussed in this paper has the same structure as the general series system. The relational model formulated by Kao and Hwang (2010) for series systems can thus be adopted for the formulation. If the two quantities ignored by Kao and Hwang (2010) are restored, it then becomes:

$$
\begin{aligned}
1/E_k^R = \min.\quad & \sum_{i=1}^{m} v_i X_{ik} + \sum_{f=1}^{g} w_f Z_{fk}^{(0)} \\
\text{s.t.}\quad & \sum_{r=1}^{s} u_r Y_{rk} + \sum_{f=1}^{g} w_f Z_{fk}^{(p)} = 1 \\
& \left(\sum_{i=1}^{m} v_i X_{ij} + \sum_{f=1}^{g} w_f Z_{fj}^{(0)}\right) - \left(\sum_{r=1}^{s} u_r Y_{rj} + \sum_{f=1}^{g} w_f Z_{fj}^{(p)}\right) \geqslant 0, \; j = 1,\ldots,n \\
& \left(\sum_{i=1}^{m} v_i X_{ij}^{(t)} + \sum_{f=1}^{g} w_f Z_{fj}^{(t-1)}\right) - \left(\sum_{r=1}^{s} u_r Y_{rj}^{(t)} + \sum_{f=1}^{g} w_f Z_{fj}^{(t)}\right) \geqslant 0, \\
& \quad j = 1,\ldots,n; \; t = 1,\ldots,p \\
& u_r, v_i, w_f \geqslant \varepsilon, \; r = 1,\ldots,s; \; i = 1,\ldots,m; \; f = 1,\ldots,g
\end{aligned} \tag{3}
$$