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An inventory control model with stochastic review interval and special sale offer

M. Karimi-Nasab [a,1], I. Konstantaras [b,*]

[a] Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran
[b] Department of Business Administration, Business Excellence Laboratory, University of Macedonia, 54006 Thessaloniki, Greece

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ABSTRACT

Periodic review inventory models are widely used in practice, especially for inventory systems in which many different items are purchased from the same supplier. However, most of periodic review models have assumed a fixed length of the review periods. In practice, it is possible that the review periods are of a random (stochastic) length. This paper presents an inventory control model in the case of random review intervals and special sale offer from the supplier. The replenishment interval is assumed to obey from two different distributions, namely, exponential and uniform distributions. Also, shortages are allowed in the term of partial backordering. For this model, its convexity condition is discussed and closed form solutions are proposed.

1. Introduction and literature review

General purpose of inventory planning models is to determine as their main decision variables (a) the time of placing an order, and (b) the order size, while minimizing total cost of the system. However, there are many extensions that search for determining further decision variables or use different objective functions. Periodic review inventory models are a major portion of the literature. One fundamental assumption about periodic systems is that the review periods are of fixed length. In practice, however, the review periods may be of a variable length. For several reasons this fact implies variations in supply time, in consumption, delays in transportations and so forth. This setting represents the real life cases where a supplier visits a retailer with random inter arrival times and the retailer replenishes his inventories based on a replenish-up-to-level inventory control policy. Readers should note that "review interval" (or review period) and "replenishment interval" are not exactly the same. To be precise, a review of inventory is only an opportunity for replenishment. So for example, with fixed review intervals, replenishment does not necessarily take place at fixed intervals. On this basis, many scholars use the term "replenishment" only when referencing to actual replenishment decisions, and elsewhere using "review". However, in a case that replenishment is done just when the agent of the supplier visits a retailer, one can use both terms alternatively without any problem, but we follow the previous researchers' convention through out this paper.

It is well known that under general assumptions waiting times must be exponential or gamma for many inventory systems. On the other hand, limited time price incentives such as a one-time-only or temporary price reduction are frequently encountered in procurement and retailing. Manufacturers and suppliers offer such incentives to their customers or retailers for various reasons such as a need to reduce excess inventories or to take up the slack in their production facilities thereby encouraging consumers to order larger than usual quantities at a discounted price [21]. Special sale offer is an ordinary issue for some industries such as medicine and landing [24]. In this paper, both of these cases are considered simultaneously and the review interval is assumed to obey from two different distributions, namely, exponential and uniform distributions, while in a cycle among others we are encountered with special sale offer from the supplier.

Now, a brief chronological review of the literature is reported. Goyal [11] stated that the models currently available in the literature for determining the special order quantity as well as for determining the benefit of placing a special order are not flawed as far as the EOQ approach to modeling inventory decisions is concerned. Wee and Yu [25] developed an EOQ model for exponentially decaying inventory under temporary price discount. Luciano and Peccati [17] discussed the use of adjusted present value techniques in a problem of inventory management with the temporary sale price, in the presence of equity or debt financing. Arcelus et al. [5] assumed the increasing saliency of special offers as a sales promotion tool, and analyzed the advantages and disadvantages of the two most common payment reduction schemes, namely a decrease in the purchase price and a delay in the payment of the merchandise. Following some of the latest empirical evidence in the sales promotion field, the model includes a price-dependent demand, where price incorporates the ability of the retailer to pass on some

* Corresponding author. Tel./fax: +30 2651067662.
E-mail addresses: mehdikariminasab@iust.ac.ir (M. Karimi-Nasab), ikonst@uom.gr (I. Konstantaras).
[1] Tel.: +98 9127990527.

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of the savings to the customers. Then, Abad [1] analyzed the problem for when the promotion lasts a finite time interval. He considered two cases: (1) using technology (e.g. point-of sale systems), the supplier can insure that the discount is applicable to only units resold during the promotion, and (2) the discount is applicable to units purchased during the promotion.

Similarly, Arcelus et al. [3] examined a retailer's response to a vendor's trade promotion, which is guaranteed to last some fixed length of time, followed by an additional (uncertain) period of time, "while supplies last". They developed a general-special-sales model under uncertainty for maximizing profit of a retailer during both the deterministic and the stochastic portions of the special sale period. They showed that the total profit for the retailer's response problem may be written as a function only of the ending inventory of the deterministic period. Arcelus et al. [6] modeled the retailer's profit-maximizing retail promotion strategy, when confronted with a vendor's trade promotion offer of credit and/or price discount on the purchase of regular or perishable merchandise. They determined (i) the size of the special order to be placed from the vendor, under the different types of possible trade incentives offered; (ii) the price and/or credit-terms incentives to be passed onto its own customers to stimulate demand on a temporary basis; and (iii) the quantity to be sold under these one-time-only conditions.

Ertogral and Rahim [10] considered an inventory planning problem when there are periodic reviews but the replenishment intervals are taken to be random variables, while ordering based on a replenish-up-to-level inventory control policy. They also assumed that only a certain fraction of unmet demand is backordered and the rest of it is lost. Martınez-Ruiz et al. [18] used a semi-parametric regression approach to model the complex nature of the problem. Their model is developed at the brand level using daily store-level scanner-data, which allows the study of several promotional effects, such as the influence of the day of the week both in promotional and non-promotional periods. Sarker and Al-Kindi [23] developed economic order quantity (EOQ) models with a discounted price to obtain the optimal ordering policy during the sale period for five different cases: (a) coincidence of sale period with review time, (b) non-coincidence of sale period with review time, (c) sale period is longer than a cycle, (d) discounted price as a function of the special ordering quantity, and (e) incremental discount. Their objective was to take the maximum possible advantage from the discounted price by ordering a special quantity during the sale period. Goyal and Jaber [12] suggested a marginal analysis approach to determine the optimal ordering policy for one of the models proposed by Sarker and Al Kindi [23].

Abad [2] characterized the buyer's response to a temporary price reduction. The model is applicable for a buyer whose strategy is to include transportation costs in their purchase decisions. The buyer may want direct control on his in bound logistics costs. The company may have outsourced its logistics function and as a result is charged for freight as invoiced by the public motor carrier. Chiang [7] considered a periodic review inventory model where the review intervals are random and used dynamic programming to solve the model. Liu et al. [16] considered the same problem but for perishable case. They developed a general case formulation along with providing analysis for two special cases of exponential and uniform distribution for review intervals. Arcelus et al. [4] considered a profit-maximizing retailer's decision process when anticipating a vendor's offer of a temporary sale at a reduced price.

Chu and Chung [8] and then Leung [15] provided mathematical representation for the sensitivity analysis of previous models in the case of partial backordering. However, different forms of backordering can be considered in practice, based on the case. For example, Hu et al. [13] proposed an inventory model with partial backordering and unit backorder cost linearly increasing with the waiting time, or Pentico and Drake [19] considered partial backordering in an EOQ model by a new approach. Also, disregarding the type of formulation when considering partial backordering, numerous papers have developed such models for various applications (see for example [9,22,14]). Readers are referred to Pentico and Drake [20] for a survey of deterministic models for the EOQ and EPQ with partial backordering.

This paper formulates an inventory control problem where (i) review intervals are stochastically distributed from exponential and uniform distributions (like [10,16]), (ii) replenishment is done when the inventory is reviewed, (iii) supplier offers special sale (following the works of [3,5,1]), and (iv) shortages are allowed in the term of partial backordering [8,15]. The general logic of our formulation is to maximize the gross savings from placing special order in the case of dealing with special sales. Then, convexity conditions are discussed and closed form solutions are obtained. To our best knowledge, and according to the literature reviewed, this problem is not comprehensively in the literature. The main novelty of the current paper comes back to considering working conditions (i), (ii), (iii) and (iv) simultaneously, where these settings can be confronted in many real world inventory cases such as drugstores, groceries, etc. In such cases, it is common to deal with special sale offer from supplier while the goods are received in stochastic intervals (i.e. replenishment intervals are stochastic variables).

The other parts of this paper are organized as follows: in Section 2, the mathematical formulation of the problem is given along with detailed explanations about problem assumptions and characteristics. Two special distribution cases are considered in Sections 3 and 4 gives some concluding remarks and potential future researches.

2. Model formulation

In this section, first the fundamental assumptions of the problem are reviewed. Then, the list of input parameters and decision variables are introduced. Finally, the mathematical formulation is constructed in a step by step manner. The followings are most important assumptions of the problem:

- demand rate is constant and deterministic over time,
- shortage is allowed in the term of partial backorders, i.e. a portion of the shortages between two consecutive review times is lost and the others will be transferred to the next time to be satisfied,
- the times between replenishments are i.i.d. random variables,
- regularly the system has the inventory ceiling of R, while in the case of special sales this could be changed to R_s,
- there is a unique special sales opportunity for us that we could buy much larger than regular working cycles,
- other cost parameters are some non-stochastic deterministic values.

Input parameters

D	demand rate of the product over time
t	time between two consecutive replenishment/review times ($t_{min} \leqslant t \leqslant t_{max}$)
$f_t(z)$	probability distribution function (pdf) of t
$F_t(z)$	cumulative distribution function (cdf) of t
t_{min}	minimum expected value of the length of replenishment/review time (i.e. $t_{min} \leqslant t$)
t_{max}	maximum expected value of the length of replenishment/review time (i.e. $t \leqslant t_{max}$)
α	percent of the shortages that are backordered