A Simple Discrete-Time Tracking Differentiator and Its Application to Speed and Position Detection System for a Maglev Train

Hehong Zhang, Yunde Xie, Gaoxi Xiao, Chao Zhai, and Zhiqiang Long

***Abstract*— In this brief, a novel tracking differentiator (TD) based on discrete time optimal control (DTOC) is presented. In particular, using the state back-stepping method, a DTOC law for a discrete-time, double-integral system is determined by linearized criterion, which equips the TD with a simple structure. The analysis of the proposed TD reveals its filtering mechanism. Simulation results show that it performs well in signal tracking and differentiation acquisition, and reduces the computational resources needed. Experiments conducted on the speed and position detection system for a maglev train demonstrate that the proposed TD group, with moving average algorithm, can filter noises, amend distortion signals effectively, and compensate for phase delays when the train is passing over track joints.**

***Index Terms*— Discrete time, filter, linearized criterion, maglev train, phase delay, time optimal control (TOC), tracking differentiator (TD).**

I. Introduction

THE differentiation of a given signal in real time is a well known yet challenging problem in control engineering and theory [1], [2]. The proportional—integral—derivative (PID) control law developed in the last century still plays an essential role in modern control engineering practice [3], [4]. However, since derivative signals are prone to corruption by noise and derivative control is usually not physically implementable, the PID control is usually degraded to PI control [5]. To deal with this, researchers have proposed many different approaches for differentiator design, including those based on a high-gain observer [6], a linear time-derivative tracker [7], a super-twisting second-order sliding mode algorithm [8], robust exact differentiation [9], [10], a finite time convergent differentiator [11], and so on.

Initially proposed by Han [12], a noise-tolerant time optimal control (TOC)-based tracking differentiator (TD) allows one to avoid a setpoint jump in the emerging active disturbance rejection controller. The advantage of this TD is that it sets a weak condition on the stability of the systems to be constructed for TD and requires a weak condition on the input. In addition, it also has the advantage of maintaining a greater level of smoothness compared to the chattering problem encountered by sliding mode-based differentiators [13]. The following presents a brief outline for the construction of this TD.

The double-integral system is defined as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \quad |u| \le r \end{cases} \tag{1}$$

where r is a constant. Note that, depending on the physical limitations of each application, the parameter r can be selected accordingly to speed up or slow down the transient profile. The resulting feedback control law that drives the state from any initial point to the origin in the shortest time is [14], [15]

$$u = -r\,\mathrm{sign}\left(x_1 - \upsilon + \frac{x_2|x_2|}{2r}\right) \tag{2}$$

where υ is the desired value for x_1. The switching curve function is $\Gamma(x_1, x_2) = x_1 + (x_2|x_2|/2r)$. Using this principle, we can obtain the desired trajectory and its derivative by solving the following differential equations:

$$\begin{cases} \dot{\upsilon}_1 = \upsilon_2, \\ \dot{\upsilon}_2 = -r\,\mathrm{sign}\left(\upsilon_1 - \upsilon + \dfrac{\upsilon_2|\upsilon_2|}{2r}\right) \end{cases} \tag{3}$$

where υ_1 is the desired trajectory and υ_2 is its derivative.

With the developments in computer control technology, most control algorithms are now implemented in the discrete time domain. Direct digitization of a continuous TOC solution of (2) is problematic in practice because of the high-frequency chattering of the control signals [16]. This problem can be addressed by using a discrete-time solution for a discrete double-integral system $\upsilon_1(k+1) = \upsilon_1(k) + \mathrm{h}\upsilon_2(k)$, $\upsilon_2(k+1) = \upsilon_2(k) + \mathrm{h}u(k)$, $|u(k)| \le r$ to obtain $u = \mathrm{Fhan}(\upsilon_1(k) - \upsilon(k), \upsilon_2(k), r_0, h_0)$, where h is the sampling period and r_0 and h_0 are the controller parameters [12], [16].

However, the discrete TOC (DTOC) law (Fhan) of the TD is determined by comparing the position of the initial state with the isochronic region obtained through nonlinear boundary transformation. This makes the structure of a TD to be complex with nonlinear calculations, including square root

Manuscript received July 20, 2017; revised February 28, 2018; accepted April 26, 2018. Manuscript received in final form April 27, 2018. This work was supported in part by the National Research Foundation of Singapore through the Campus for Research Excellence and Technological Enterprise Program under the Future Resilient System Project at the Singapore-ETH Center and in part by the Ministry of Education, Singapore under Contract MOE 2016-T2-1-119. The work of H. Zhang was supported by Interdisciplinary Graduate School, Nanyang Technological University, Singapore. Recommended by Associate Editor A. Zolotas. *(Corresponding author: Yunde Xie.)*

H. Zhang is with Interdisciplinary Graduate School, Nanyang Technological University, 639798 Singapore and also with the College of Mechatronics Engineering and Automation, National University of Defense Technology, Changsha 410073, China (e-mail: hzhang030@e.ntu.edu.sg; hehongzhangnudt@hotmail.com).

Y. Xie is with Beijing Enterprises Holding Maglev Technology Development Company Limited, Beijing 10024, China (e-mail: xieyunde@outlook.com).

G. Xiao and C. Zhai are with the School of Electrical and Electronic Engineering, Nanyang Technological University, 639798 Singapore (e-mail: zhaichao@ntu.edu.sg; egxxiao@ntu.edu.sg).

Z. Long is with the College of Mechatronics Engineering and Automation, National University of Defense Technology, Changsha, China (e-mail: lzq@maglev.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCST.2018.2832139

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

calculations. In this brief, the mathematical derivation of a new closed-form DTOC law for discrete form of the system in (1) is presented. Unlike the control law Fhan, the DTOC law is based on a linearized criterion that depends upon the position of the initial state point on the phase plane. In doing so, the new control law has a simpler structure that is much easier to be applied in practical engineering scenarios. Experiments are carried out on position signal processing for position sensing in the speed and position detection system of a maglev train [17], [18]. In practice, the signals from the position sensor may be aberrant due to the existence of track joints, which leads to low efficiency of the traction system or even safety misadventures. The proposed TD can be used to construct a TD group with a moving average algorithm that filters the noises, compensates for phase delays, and amends distortion signals when the train is passing over the track joints.

This brief is organized as follows: the new DTOC law is proposed in Section II. The structure of the TD and its filtering characteristic are discussed in Section III. In Section IV, numerical simulation results are presented to compare the performance of signal tracking, differentiation acquisition and the computational resources needed in field-programmable gate array (FPGA) application between the control law Fhan and the proposed one, followed by experimental results on position signal processing for the speed and position detection system of a maglev train. Finally, Section V concludes this brief.

II. Discrete Time Optimal Control Law

Consider a discrete-time double-integral system

$$x(k+1) = Ax(k) + Bu(k), \quad |u(k)| \leq r \tag{4}$$

where

$$A = \begin{pmatrix} 1 & h \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ h \end{pmatrix}$$

and $x(k) = [x_1(k), x_2(k)]^T$. The objective here is to derive a TOC law directly in discrete time domain. The problem is defined as follows.

DTOC Law: Given system (4) and its initial state $x(0)$, determine the control signal sequence, $u(0), u(1), \ldots, u(k)$, such that the state $x(k)$ is driven back to the origin in a minimum and finite number of steps, subject to the constraint of $|u(k)| \leq r$. That is, finding $u(k^*)$, $|u(k)| \leq r$, such that $k^* = \min\{k | x(k+1) = 0\}$.

In bang-bang control, the control signal switches between its two extreme values ($u = +r$ or $u = -r$) around the switching curve, and it switches the sign instantaneously after reaching the switching curve. For a discrete time system, however, the process of sign-switching occurs within a sampling period h. During that process, the corresponding state sequences remain in a certain region (denoted as Ω) near the switching curve. The control signals for the state sequences in the region Ω are determined by a linearized criterion. The control signal varies from a certain positive (negative) value to a negative (positive) value when control signal u passes from one side of the region Ω to the other. All initial state sequences outside region Ω when the control signal takes an extreme value, i.e., $u = +r$ or $u = -r$, are located at certain curves, referred to as boundary curves Γ_A and Γ_B. Region Ω is surrounded by these boundary curves. In addition, when the value of the control signal varies in $[-r, r]$, there exists a state that corresponds to $u = 0$. All states that correspond to $u = 0$ constitute another curve, which is referred to as the control characteristic curve Γ_C.

In deriving the DTOC law, one must find the control signal sequence for any initial state point $x(0) \in \Omega$ or $x(0) \notin \Omega$. The whole task is divided into two parts as follows.

I: Determine the boundary curves of region Ω and the control characteristic curve based on the state back-stepping approach, i.e., the representation of the initial condition $x(0) = [x_1(0), x_2(0)]^T$ in terms of h and r, from which the state can be driven back to the origin in $(k+1)$ steps.

II: For any given initial condition $x(0) \in \Omega$ or $x(0) \notin \Omega$, find the corresponding control signal sequence.

A. Determination of the Boundary Curves and Control Characteristic Curve

For any initial state sequence, at least one admissible control sequence exists, e.g., $u(0)$, $u(1)$, $\ldots$, $u(k)$, that makes the solution to (4) satisfy $x(k+1) = 0$. Under the initial condition $x(0)$, the solution is

$$x(k+1) = A^{k+1}x(0) + \sum_{i=0}^{k} A^{k-i}Bu(i) \tag{5}$$

where $x(0) = [x_1(0), x_2(0)]^T$ and $i = 0, 1, 2, \ldots, k$. It manifests that $x(k+1) = 0$. Therefore, the initial condition satisfies

$$x(0) = \sum_{i=0}^{k} \begin{pmatrix} (i+1)h^2 \\ -h \end{pmatrix} u(i). \tag{6}$$

Adopting the state back-stepping approach, discussed earlier, we can determine the two boundary curves Γ_A and Γ_B as well as the control characteristic curve Γ_C as follows.

To obtain the boundary curve Γ_A, we suppose that $\{a_{+k}\}$ and $\{a_{-k}\}$ are the sets of any $x(0)$ that can be driven back to the origin with the control signal sequence $u(i) = +r$ or $u(i) = -r$, $i = 0, 1, 2, \ldots, k$. For this we specify that all initial states in set $\{a_{+k}\}$ consist of Γ_A^+ and all initial states in set $\{a_{-k}\}$ consist of Γ_A^-.

For set $\{a_{+k}\}$, the following result holds when the control signal sequence takes on $u(i) = +r$ according to (6):

$$x(0) = r\sum_{i=0}^{k} \begin{pmatrix} (i+1)h^2 \\ -h \end{pmatrix}. \tag{7}$$

And we have $x_1(0) = \mathrm{rh}^2((k^2/2) + (3k/2) + 1)$ and $x_2(0) = -\mathrm{rh}(k+1) < 0$. Simplifying $x(0)$ into x and eliminating the variable k results in the boundary curve Γ_A^+, which is $x_1 = (x_2^2/2r) - (1/2)\mathrm{hx}_2$, where $x_2 < 0$. Similarly, we can get the boundary curve Γ_A^-: $x_1 = -(x_2^2/2r) - (1/2)\mathrm{hx}_2$, where $x_2 > 0$. Therefore, the entire boundary curve Γ_A (see Fig. 1) is

$$\Gamma_A : x_1 + \frac{x_2|x_2|}{2r} + \frac{1}{2}\mathrm{hx}_2 = 0. \tag{8}$$