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Stability and monotonicity in newsvendor situations

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ABSTRACT

This study considers a supply chain that consists of n retailers, each of them facing a newsvendor problem, and a supplier. Groups of retailers might increase their expected joint profit by joint ordering and inventory centralization. However, we assume that the retailers impose some level of stock that should be dedicated to them. In this situation, we show that the associated cooperative game has a non-empty core. Afterwards, we concentrate on a dynamic situation, where several model cost parameters and the retailers' dedicated stock levels can change. We investigate how the profit division might be affected by these changes. We focus on four monotonicity properties. We identify several classes of games with retailers, where some of the monotonicity properties hold. Moreover, we show that pairs of cooperative games associated with newsvendor situations do not necessarily satisfy these properties in general, when changes in dedicated stock levels are in concern.

1. Introduction

In this paper, we consider a distribution system that consists of a supplier and n independent retailers, each facing a stochastic demand. Each retailer solves a single period problem (newsvendor problem), i.e., at the start of the period, every retailer determines his order quantity that maximizes his expected profit anticipating that, after the products are delivered to the retailers, demands are realized and satisfied from the stock as much as possible. In this network, we study the inventory pooling coalitions in which the retailers can jointly invest in a common pool of inventory to be allocated after demand realization. In a specific cooperation scenario, we study the stability of these coalitions in static and dynamic settings.

Benefits of inventory pooling, i.e., cost savings and profit increase, have been studied in different inventory settings (Eppen, 1979; Eppen and Schrage, 1981; Chen and Lin, 1989; Chang and Lin, 1991; Cherikh, 2000). These early studies assume single ownership of the system. Individual firms, however, are especially interested in what they can get for themselves from inventory centralization. Several other papers have investigated the allocation of benefits (reduced cost or increased profit) problem and proposed several mechanisms. For instance, Gerchak and Gupta (1991) compared four simple allocation mechanisms and showed that only one of them guarantees lower cost for every store than its stand-alone cost. Robinson (1993) extended their analysis to other allocation mechanisms, i.e., the Shapley value (cf. Shapley, 1953) and the Lounderback allocation (Lounderback, 1976). Hartman and Dror (1996) examined allocation mechanisms for this setting using three criteria. These are core non-emptiness, computational ease and justifiability. The core concept, a measure of stability, has also received special interest by several other papers and the core non-emptiness has been shown for different newsvendor settings: newsvendors with a common pool of inventory (Hartman et al., 2000; Müller et al., 2002; Slikker et al., 2001), and newsvendors with lateral transshipment or multiple channels of supply (Slikker et al., 2005; Özen et al., 2008; Chen and Zhang, 2009). All of these studies assume complete pooling of inventory, i.e., inventory can be diverted to satisfy demand that creates the highest profit from any stock point. However, the benefits of pooling of stock can also be seen in restrictive settings. Anupindi et al. (2001) considered a distribution system where the retailers keep local inventory. After satisfying their local demand, the retailers cooperate by transshipping excess inventory in one location to satisfy excess demand in another location. They derived a profit sharing mechanism based on dual prices of the optimal shipping problem after demand realization, which is a core element and leads to joint optimal orders being an equilibrium. The model of Anupindi et al. (2001) is extended in several directions by Granot and Sošić (2003) and Sošić (2006).

In this paper, we do not consider a complete consolidation of inventories when the retailers cooperate. Instead, we assume that the retailers invest in a common pool of inventory but each retailer asks a minimum amount of inventory to be dedicated for him,

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which will be utilized if the demand in his market appears to be good. However, in case of low demand realization, the retailers release not needed inventory for other retailers' use. There could be several possible reasons for the retailer to impose such minimum level of dedicated inventories:

- Ensure income from operations: In a high selling session, it is a main tendency of a cooperation to provide the more profitable markets with the majority of the available goods in order to increase total system profit. This behavior may leave the other markets with insufficient stock. To survive in the local market and preserve marketing strength, a retailer may want to stay active in the industry. A dedicated quantity guarantees the retailer to receive an income from the business to support his inside operations (instead of being compensated only at the end of the selling period) and continue to be active in the market.
- Ensure local competitive power: The retailers might be in quantity competition in their local market and require a certain level of dedicated inventory to stay competitive.
- Ensure some customer service level: Another important factor in surviving in the market is customer satisfaction. Using minimum level of dedicated inventory, the retailer can ensure a reasonable customer service level.

We are first interested in the stability of this type of cooperation and focus on the core concept as many papers in the literature (see Hartman et al., 2000; Müller et al., 2002; Slikker et al., 2001; Slikker et al., 2005; Özen et al., 2008). The core concept considers a natural criterion for stability that is each retailer should do better in the coalition than pursuing any of their alternatives, i.e., working alone or forming another coalition. In this research, we work with the core concept. However, we carry the stability notion a step further and we are interested in a stability measure that considers the effects of changes in the environment. In our case, the retailers in such coalitions will be interested in the form of the cooperation when a change in the environment occurs. The retailers should feel that they are not discriminated or deceived under such a situation. In other words, a new core distribution of the expected total profit that does not discriminate against any of the retailers with respect to the original profit division is desired. The core is a strong concept that ensures stability in the given framework, however has less to say when there is a different framework following the original. Note that distributing the total profit using a core element is strong enough for the stability of the cooperation in the changed situation as well. However, we would like to analyze some further fairness criteria, which the retailers would naturally consider knowing the new division of total profit. Even in the situation where these fairness criteria are hard to satisfy, developing an understanding is important for the continuation of the close relations, which is critical for coordinated decision making and, hence, for the success of cooperation.

In general, monotonicity notions from cooperative game theory can be used to address this issue. Several papers study monotonicity in TU-games. Megiddo (1974) and Young (1985) studied aggregate monotonicity and coalitional monotonicity, respectively. Young (1985) also showed that no core allocation mechanisms can be coalitionally monotonic on coalitional games with 5 or more players. Afterwards Housman and Clark (1998) extended this result to games with 4 players. Sasaki (1995) and Nunez and Rafels (2002) analyzed monotonicity in assignment games. None of the monotonicity properties studied above, however, covers the cases that we analyze in this paper. Ichiishi (1981) introduced three welfare criteria on the core of the games and he established necessary and sufficient conditions for the criteria to be satisfied. Two of those criteria represent a fairness argument we like to study in this paper. We discuss this issue in more detail when we introduce four monotonicity properties in Section 2.2.

The outline of the paper is as follows: Section 2.1 gives preliminaries on cooperative game theory. In Section 2.2, we introduce 4 monotonicity properties and derive several sufficient conditions. Section 3.1 introduces the newsvendor situations with dedicated stock and the associated cooperative games. Moreover, we focus on the existence of stable profit distributions, which is shown by proving that these games have non-empty cores. In Section 3.2, we investigate the cases, where the retailers' parameters for cooperation are changed, e.g., changes in the dedicated stock levels, selling prices, purchasing cost and penalty cost, which affect the outcome of the coalition. We identify two types of changes. In the first one, all retailers' parameters are changed, and in the latter single retailer's parameters are changed. We focus on the issue of whether we can find a core distribution of total profit for the new situation, which does not discriminate against any of the retailers. This issue is captured by the 4 monotonicity properties introduced in Section 2.2. In Section 3.2.1, we identify several classes of newsvendor games where two of the monotonicity properties hold regarding the changes in selling price, purchasing cost and penalty cost. In Section 3.2.2, we analyze the monotonicity properties under changes in retailers' dedicated stock levels. After providing examples that none of the properties are guaranteed to hold for cooperative games associated with newsvendor situations, we focus on a class of newsvendor games for which one of the monotonicity properties holds. We conclude our paper in Section 4 with final remarks. The proofs that are not presented in the main body of the paper can be found in the online appendix.

2. Preliminaries and monotonicity

2.1. Preliminaries

In this section, we give a brief introduction to cooperative game theory and introduce some notation. Let N be a finite set of players, $N = \{1,\ldots,n\}$. A subset of N is called a *coalition*. A function v, assigning a value $v(S)$ to every coalition $S \subseteq N$ with $v(\emptyset) = 0$, is called a *characteristic function*. The value $v(S)$ is interpreted as the maximum total profit that coalition S can obtain through cooperation. Assuming that the benefit of a coalition S can be transferred among the players of S, a pair (N, v) is called a *cooperative game with transferable utility* (TU-game). For a game (N, v), $S \subset N$ and $S \neq \emptyset$, the *subgame* $(S, v_{|S})$ is defined by $v_{|S}(T) = v(T)$ for each coalition $T \subseteq S$.

In reality, the players are not primarily interested in benefits of a coalition but in their individual benefits that they make out of that coalition. A division is a *payoff vector* $y = (y_i)_{i\in N} \in \mathbb{R}^N$, specifying for each player $i \in N$ the benefit y_i. A division y is called *efficient* if $\sum_{i\in N} y_i = v(N)$ and *individually rational* if $y_i \geqslant v(\{i\})$ for all $i \in N$. Individual rationality means that every player gets at least as much as what he could obtain by staying alone. The set of all individually rational and efficient divisions constitutes the *imputation set*:

$$I(v) = \left\{ y \in \mathbb{R}^N \mid \sum_{i\in N} y_i = v(N) \quad \text{and} \quad y_i \geqslant v(\{i\}) \quad \text{for each } i \in N \right\}.$$

If these rationality requirements are extended to all coalitions, we obtain the *core*:

$$Core(v) = \left\{ y \in \mathbb{R}^N \mid \sum_{i\in N} y_i = v(N) \quad \text{and} \quad \sum_{i\in S} y_i \geqslant v(S) \quad \text{for each } S \subseteq N \right\}.$$

Thus, the core consists of all imputations in which no group of players has an incentive to split off from the grand coalition N and form a smaller coalition, because they collectively receive at least as much as what they can obtain by cooperating on their own. Note that the core of a game can be empty.