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Multi-tier binary solution method for multi-product newsvendor problem with multiple constraints

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ABSTRACT

This paper considers a multi-product newsvendor problem with multiple constraints. Multiple constraints in the problem make it more challenging to solve. Previous research has attempted to solve the problem by considering two-constraint case or/and using approximation techniques or active sets methods. The solution methods in literature for solving multi-constraint problem are limited or cumbersome. In this paper, by analyzing structural properties of the multi-constraint multi-product newsvendor problem, we develop a multi-tier binary solution method for yielding the optimal solution to the problem. The proposed method is applicable to the problem with any continuous demand distribution and more than two constraints, and its computational complexity is polynomial in the number of products. Numerical results are presented for showing the effectiveness of our method.

1. Introduction

Multi-product constrained newsvendor problem is a classical inventory management problem, in which multiple products are procured before random demand realization and they are sold during a selling season. The newsvendor's objective is to determine order quantity of each product for optimizing profit or cost subject to budget or other resource constraints. Hadley and Whitin (1963) firstly studied the multi-product newsvendor problem with a single budget and specific demand distribution. After their seminal work, many researchers investigated various extensions and applications of multi-product newsvendor problem. Main extensions include dealing with general demand distributions (Lau and Lau, 1996; Moon and Silver, 2000; Abdel-Malek et al., 2004; Zhang et al., 2009), various objective functions (Gallego and Moon, 1993; Vairaktarakis, 2000; Choi et al., 2011; Choi and Ruszczyński, 2011), different demand patterns (Shaha and Avittathur, 2007; Zhang, 2011), and complex supply structures, such as random yield (Yang et al., 2007), preseason production (Chung et al., 2008), outsourcing production (Zhang and Du, 2010), dual sourcing with portfolio contract (Zhang and Hua, 2010), supplier discount (Zhang, 2010) and reservation policy (Chen and Chen, 2010).

One of these important extensions is to investigate the multi-product newsvendor problem with general demand distribution and multiple constraints. Considering the difficulty in solving multi-constraint problem, many researchers have attempted to solve different versions of single-constraint model. Gallego and Moon (1993) modeled and solved the singly constrained problem in which demand distribution is unknown and only the first two moments are given. Erlebacher (2000) studied the single-constraint model with similar cost structures or uniform demand distribution. Vairaktarakis (2000) investigated some min–max regret formulations for single-constraint model. Abdel-Malek et al. (2004) proposed a generic iterative method, which can generate the optimal solution for uniform demand distribution, and near optimal solution for other continuous distributions. As Lau and Lau (1996) and Abdel-Malek and Montanari (2005a,b) noticed, some of the existing research ignored the non-negativity constraints, which could lead to negative order quantities. To deal with non-negativity constraints, Abdel-Malek and Montanari (2005a) developed a modified generic iterative method by analyzing the solution space of the single-constraint problem, and their approach can produce exact and approximate solutions for continuous demand distributions. Recently, Zhang et al. (2009) proposed a binary solution method for the single-constraint model with non-negativity constraints, which yields exact solution to the problem with any continuous demand distribution.

Since newsvendor-type firms often face the situation of multiple resource constraints, such as budget, capacity, volume and space restrictions, it is useful to model and solve multi-constraint problem directly. Unfortunately, it is difficult to extend the solution methods for the single-constraint models to solve multi-constraint problem. Several researchers have attempted to solve multi-constraint problem by considering different cases. Ben-Daya and Raouf (1993) developed a Lagrange based method for solving two-constraint problem with uniform demand distribution. Abdel-Malek and Montanari (2005b) developed a generic iterative

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method to solve two-constraint model by examining the dual of the solution space as defined by the two constraints, which finds the optimal or near optimal solution. These methods are difficultly extended for solving the problem with more than two constraints. Lau and Lau (1995, 1996) studied multi-constraint problem and developed heuristics based on active sets methods for an efficient search across Lagrange multipliers. Their method becomes cumbersome when solving the problem with more than one constraint. Recently, Niederhoff (2007) solved multi-constraint problem with general demand distribution by approximating the objective function with piecewise linear interpolate, and she showed that close approximate solution can be found using convex separable programming. Abdel-Malek and Areeratchakul (2007) solved multi-constraint problem with general demand distribution by approximating the objective function with quadratic function, and they reported that their method yields less than 1% error in the optimum value of the objective function.

Previous research in literature solved the multi-constraint multi-product newsvendor problem by considering two-constraint case, limiting the type of demand distribution, or using approximation techniques or active sets methods. These solution methods are limited or cumbersome when solving the problem with more than two constraints. In this paper, we revisit the multi-constraint multi-product newsvendor problem with general demand distribution and non-negativity constraints. Through analyzing the structural properties of the problem, we develop a multi-tier binary solution method based on Karush–Kuhn–Tucker (KKT) conditions, which produces the optimal solution to the problem with any continuous demand distribution. Numerical results are presented for demonstrating the effectiveness of our method.

The rest of this paper is organized as follows. Section 2 describes the problem. In Section 3, the structural properties and solution method are established. Numerical results are reported in Section 4. Section 5 concludes the paper with a discussion of some future extensions. All proofs are presented in appendix.

2. Problem formulation

Before modeling the multi-constraint multi-product newsvendor problem, we first give some notation below.

N = total number of products;
M = total number of resources;
i = index for product, $i = 1,\ldots,N$;
j = index for resource, $j = 1,\ldots,M$;
k_i = per unit cost of product i;
h_i = per unit overage cost of product i;
υ_i = per unit underage cost of product i;
$c_{i,j}$ = coefficient of product i of resource j;
C_j = available amount of resource j;
D_i = random demand for product i;
$f_i(\cdot)$ = probability density (positive) function of demand for product i;
$F_i(\cdot)$ = cumulative demand distribution function for product i, $F_i(0) = 0$;
$F_i^{-1}(\cdot)$ = inverse demand distribution function for product i;
x_i = newsvendor's order quantity for product i.

In the multi-constraint multi-product newsvendor problem, the newsvendor's objective is to determine order quantity of each product for minimizing total expected cost subject to some resource constraints and non-negativity constraints. Let $(\cdot)^+ = \max\{\cdot, 0\}$ and $x = (x_1,\ldots,x_N)^T$, and denote by $E(\cdot)$ the expectation operator, then the multi-constraint multi-product newsvendor problem can be expressed as follows (denoted as problem P):

$$\text{Min} \quad \pi(x) = \sum_{i=1}^{N} \pi_i(x_i) = \sum_{i=1}^{N} \left[k_i x_i + h_i E(x_i - D_i)^+ + \upsilon_i E(D_i - x_i)^+\right], \tag{1}$$

$$\text{subject to} \quad \sum_{i=1}^{N} c_{i,j} x_i \leqslant C_j, \quad j = 1,\ldots,M, \tag{2}$$

$$x_i \geqslant 0, \quad i = 1,\ldots,N. \tag{3}$$

In problem P, $k_i x_i$ is the procuring cost of product i, $h_i E(x_i - D_i)^+$ is the expected overage cost of product i, $\upsilon_i E(D_i - x_i)^+$ is the expected underage cost of product i, $\pi_i(x_i)$ is the expected cost of product i, and $\pi(x)$ is the total expected cost. Eq. (2) specifies the resource constraints, and Eq. (3) indicates the non-negativity constraints. We include the procuring cost $k_i x_i$, $i = 1,\ldots,N$, in the objective function of problem P as the often-used model in literature does. Note that the procuring cost is excluded from the objective function of some models in literature, in that situation, the newsvendor's objective is to minimize the expected overage and underage cost, which is equivalent to maximizing the expected profit. Actually, the existence of the procuring cost in the objective function has no impact on our study, and our method is also applicable to the model without the procuring cost. We will solve an example without the procuring cost in the objective function in numerical study.

Using $(D_i - x_i)^+ = D_i - x_i + (x_i - D_i)^+$ and integration by parts, the objective function of problem P can be rewritten as:

$$\begin{aligned}
\pi(x) &= \sum_{i=1}^{N} [k_i x_i + h_i E(x_i - D_i)^+ + \upsilon_i E(D_i - x_i + (x_i - D_i)^+)] \\
&= \sum_{i=1}^{N} [\upsilon_i E(D_i) + (k_i - \upsilon_i) x_i + (h_i + \upsilon_i) E(x_i - D_i)^+)] \\
&= \sum_{i=1}^{N} [\upsilon_i E(D_i) + (k_i - \upsilon_i) x_i + (h_i + \upsilon_i) \int_0^{x_i} (x_i - z_i) dF_i(z_i)] \\
&= \sum_{i=1}^{N} [\upsilon_i E(D_i) + (k_i - \upsilon_i) x_i + (h_i + \upsilon_i) \int_0^{x_i} F_i(z_i) dz_i].
\end{aligned} \tag{4}$$

Let $\tilde{x}_i$, $i = 1, \ldots, N$, be the unconstrained optimal solution, and $\bar{x}_i$, $i = 1, \ldots, N$, be the optimal solution to problem P without the resource constraints in Eq. (2). Then $\tilde{x}_i$ can be solved by setting $\frac{\partial \pi(x)}{\partial x_i} = (k_i - \upsilon_i) + (h_i + \upsilon_i) F(x_i) = 0$. Thus, we have $\tilde{x}_i = F_i^{-1}(\frac{\upsilon_i - k_i}{h_i + \upsilon_i})$. By comparing the unconstrained optimal solution $\tilde{x}_i$ and $x_i \geqslant 0$, we have $\bar{x}_i = \max\left(F_i^{-1}\left(\frac{\upsilon_i - k_i}{h_i + \upsilon_i}\right), 0\right) = F_i^{-1}\left(\left(\frac{\upsilon_i - k_i}{h_i + \upsilon_i}\right)^+\right)$, since $F_i(0) = 0$.

3. Structural properties and solution method

In this section, we first establish some fundamental structural properties of problem P, and then we develop a simple solution method based on the structural properties for solving problem P.

3.1. Structural properties

It is well known that problem P is a convex programming problem. Since the feasible domain defined by Eqs. (2) and (3) is convex, the optimality condition for problem P can be characterized via KKT conditions.

Let $\lambda_j \geqslant 0$, $j = 1, \ldots, M$, be the Lagrange multipliers for the resource constraints in Eq. (2), and $w_i \geqslant 0$, $i = 1, \ldots, N$, be the Lagrange multipliers for non-negativity constraints in Eq. (3). Denote by $\lambda = (\lambda_1, \ldots, \lambda_M)^T$, $w = (w_1, \ldots, w_N)^T$, then the Lagrange function for problem P is

$$L(x, \lambda, w) = \sum_{i=1}^{N} \pi_i(x_i) - \sum_{j=1}^{M} \lambda_j \left(C_j - \sum_{i=1}^{N} c_{i,j} x_i\right) - \sum_{i=1}^{N} w_i x_i. \tag{5}$$