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Optimal order lot sizing and pricing with free shipping

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ABSTRACT

Companies, especially those in e-business, are increasingly offering free shipping to buyers whose order sizes exceed the free shipping quantity. In this paper, given that the supplier offers free shipping, we determine the retailer's optimal order lot size and the optimal retail price. We explicitly incorporate the supplier's quantity discount, and transportation cost into the model. We analytically and numerically examine the impacts of free shipping, quantity discount and transportation cost on the retailer's optimal lot sizing and pricing decisions. We find that free shipping can benefit the supplier, the retailer, and the end customers, and can effectively encourage the retailer to order more of the good, to the extent of ordering a few times of the optimal order lot size without free shipping. The order lot size will increase and the retail price will decrease if the supplier offers proper free shipping.

1. Introduction

Companies, especially business-to-customer (B2C) and business-to-business (B2B) firms, are increasingly offering free shipping to buyers whose order sizes exceed the free shipping quantity (FSQ). For example, the B2B companies 1800contacts.com,[1] a lenses supplier, and re-inks.com,[2] a printer ink cartridges supplier, provide free shipping for web orders over US$50 and US$45, respectively. A proper free shipping policy can induce buyers to place larger orders less frequently, which allows the supplier to cut handling, order fulfillment, and production costs, enabling it to achieve economies of scale (Zhou et al., 2009). There is even evidence that free shipping has a greater impact on buyers than price discount. A survey finds that shipping and handling costs triggers 52% of the abandonment of online shopping carts (Knowledge@Wharton, 2008).

In the face of a growing number of suppliers, especially B2B suppliers, offering free shipping to their retailers, two natural questions facing the retailer arise: (1) what are the optimal order quantity and the optimal retail price given the supplier's FSQ? (2) how do the FSQ, quantity discount, and transportation cost impact on the retailer's order sizing and pricing decisions? We set out to address these two questions and present some managerial insights in this paper.

The literature on free shipping is very sparse. Lewis et al. (2006) empirically study the impact of nonlinear shipping and handling fees on purchase incidences and expenditure decisions. They find that consumers are very sensitive to shipping charges and that shipping fees influence order incidence and basket size. Leng and Parlar (2005) present a game-theoretic analysis of a free shipping problem between a seller and a buyer in the B2B context, in which the seller as the leader first sets the FSQ, then the retailer as the follower decides the purchase quantity. They assume that the shipping cost is a continuous and smooth function of the purchase value. However, in practice, shipping cost is not smooth and is even discontinuous in the purchase value (Abad and Aggarwal, 2005; Ertogral et al., 2007; Ertogral, 2008; Russell and Krajewski, 1991; Swenseth and Godfrey, 2002). Zhou et al. (2009) examine the problem of a stochastic inventory system with the free shipping option using stochastic dynamic programming. They present the optimal policy for the single-period inventory system and a heuristic policy for the multi-period case. Yang et al. (2006) consider a free shipping problem in the B2C context. They investigate the optimal shopping policy for a shopper who repeatedly purchases a non-durable product from an e-tailer and examine the e-tailer's endogenous choices of price and free shipping given rational shopper behaviour. Zhou et al. (2009) and Yang et al. (2006) assume that the e-tailer charges a fixed fee K for shipping, which is independent of the order value, if the shopper's expenditure is lower than the FSQ. In B2C transactions, it is reasonable to assume that the shipping cost is a fixed fee independent of the order value since a customer's order quantity is relative small. However, the assumption is not suitable for a supplier-retailer system or B2B transactions because the order quantities are usually very large and the quantity dispersion of

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[1] http://www.1800contacts.com/.
[2] http://www.re-inks.com/.

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the orders is usually large. Leng and Becerril-Arreola (2010) examine an online retailers' joint pricing and contingent free-shipping decisions in B2C transactions, under the assumption that the shipping fee is a linear function of the purchase amount. Hua et al. (2010) investigate the optimal order strategy of a retailer that faces deterministic or stochastic demand when suppliers offer free shipping. Hua et al. (2012) examine the newsvendor's optimal order quantity and optimal selling price when it faces stochastic demand and the free shipping option is given. They consider different ways in which price affects the demand distribution.

All the above literature except for Hua et al. (2012) does not capture quantity discounts from suppliers and freight discounts from shippers (Leng and Parlar (2005) imply freight discount, but their shipping cost is a continuous and smooth function of the purchase value), which are commonly encountered by retailers. There are plentiful studies of purchase decisions incorporating quantity discount and shipping cost, e.g., Tersine and Barman (1991), Burwell et al. (1997), Russell and Krajewski (1991), Swenseth and Godfrey (2002), Chan et al. (2002), Abad and Aggarwal (2005, 2006), and Ertogral et al. (2007).

Leng and Parlar (2005) and Zhou et al. (2009) do not incorporate pricing decisions into their models. In fact, the free shipping policies of suppliers have a significant impact on the optimal pricing decisions of their retailers. In this paper we address a new free shipping problem in the B2B context. Different from the existing studies on free shipping in the literature, we simultaneously determine the retailer's optimal order lot size and the retail price. We also incorporate both quantity and freight discounts into the model. Our transportation cost function is very general, which includes all the transportation cost functions used in the above studies, except Leng and Parlar (2005), as special cases. We analytically and numerically examine the impacts of free shipping, quantity discount, and transportation cost on the retailer's optimal lot sizing and pricing decisions. We find that free shipping can benefit the supplier, the retailer, and the end customers, and can effectively encourage the retailer to order more of the good, to the extent of ordering a few times of the optimal order lot size without free shipping.

This paper is organized as follows: in Section 2 we give a brief review of quantity and freight discounts. In Sections 3 we formulate the free shipping problem mathematically, and examine analytically and numerically the optimal lot sizing and pricing decisions. Finally we conclude the paper and suggest topics for future research in Section 4.

2. Quantity discount and transportation cost

To encourage retailers to order more goods, suppliers usually offer quantity and freight discounts to their retailers, which allow both suppliers and retailers to achieve economies of scale. The discount schedules examined in the literature are either all-unit quantity discount or incremental quantity discount. Following Hua et al. (2012), we assume in this paper the quantity discount is of the all-unit type.

The price schedule for all-unit quantity discount is as follows:

$$w(Q) = \begin{cases} w_0, & P_0 \leqslant Q < P_1, \\ w_1, & P_1 \leqslant Q < P_2, \\ \cdots & \cdots \\ w_m, & Q \geqslant P_m, \end{cases} \tag{1}$$

where $w(Q)$ is the unit purchase price if a retailer orders Q units of the good, $P_1 < P_2 < \cdots < P_m$ is the sequence of threshold quantities at which price-breaks occur, P_0 is the minimum order quantity that the supplier will accept, and $w_0 > w_1 > \cdots > w_m$ is the sequence of unit purchase prices applicable to orders whose order quantities fall in the corresponding ranges.

Since transportation cost can be upwards of 50% of the logistics cost (Swenseth and Godfrey, 2002), Carter and Ferrin (1996) advocate that transportation cost should be explicitly considered in purchase decisions. In practice, shipping service providers usually offer freight rate discounts to customers, which are similar to quantity discounts but are usually based on weight, volume, carload-lot, or standard container size that applies to units of a single product. Freight discount schedules can be classified as either all-unit freight discount or incremental freight discount. In this paper we focus on a single product and assume that the freight discount is of the all-unit type.

The transportation cost with all-unit freight discount is given as follows:

$$FC(Q) = \begin{cases} c_0Q, & P_0 \leqslant Q < P_1, \\ c_1Q, & P_1 \leqslant Q < P_2, \\ \cdots & \cdots \\ c_nQ, & Q \geqslant P_n, \end{cases} \tag{2}$$

where $FC(Q)$ is the transportation cost if the retailer orders Q units of the good (Ertogral et al., 2007) and $c_0 > c_1 > \cdots > c_n$ is the sequence of unit transportation costs. Fig. 1(a) illustrates this cost structure.

Obviously, the retailer may benefit from artificially over-declaring the shipment lot size (Chan et al., 2002; Ertogral et al., 2007). For example, the retailer will incur less transportation cost if it over-declares any shipment size in $[\alpha_0, P_1)$ as P_1 units. Thus, the transportation cost with over-declaration is given by

$$FC(Q) = \begin{cases} c_0Q, & P_0 \leqslant Q < \alpha_0, \\ c_1P_1, & \alpha_0 \leqslant Q < P_1, \\ c_1Q, & P_1 \leqslant Q < P_2, \\ \cdots & \cdots \\ c_nQ, & Q \geqslant P_n. \end{cases} \tag{3}$$

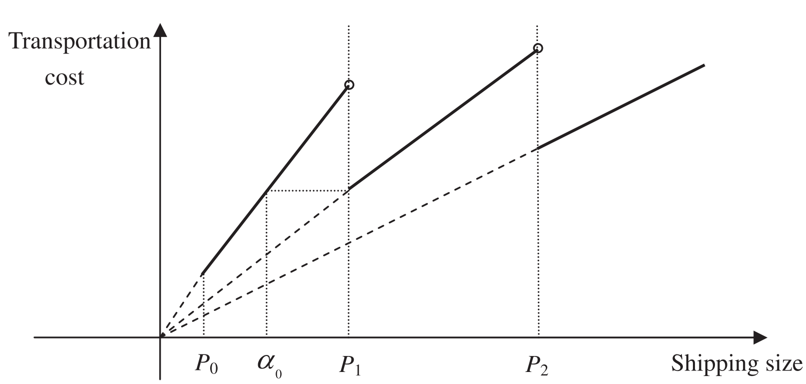

Fig. 1. Transportation cost without or with the over-declaration option.