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Innovative Applications of O.R.

A school bus scheduling problem

Byung-In Kim [a],*, Seongbae Kim [b], Junhyuk Park [b]

[a] Department of Industrial and Management Engineering, Pohang University of Science and Technology (POSTECH), Pohang, Kyungbuk 790-784, Republic of Korea
[b] Institute of Information Technology, Inc., Magnolia, TX 77354, USA

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ABSTRACT

This paper introduces a school bus scheduling problem wherein trips for each school are given. A trip consists of a sequence of bus stops and their designated school. Each school has its fixed time window within which trips should be completed. A school bus can serve multiple trips for multiple schools. The school bus scheduling problem seeks to optimize bus schedules to serve all the given trips considering the school time windows. We first model the problem as a vehicle routing problem with time windows (VRPTW) by treating a trip as a virtual stop. Two assignment problem based exact approaches are then proposed for special cases and a heuristic algorithm is proposed for more general cases. Benchmark problems and computational experiments are presented. Computational experiments show the effectiveness of the proposed approaches.

1. Introduction

This paper considers a school bus scheduling problem wherein trips for each school are given. A trip consists of a sequence of bus stops and their designated school. The trip's required service time is based on the bus stops, the number of students, and the school in a trip. Each school has its fixed time window within which trips should be completed. A school bus can serve multiple trips for multiple schools. The school bus scheduling problem presented in this paper seeks to optimize bus schedules to serve all the given trips considering the school time windows.

Fig. 1 shows an example of the school bus scheduling problem. Let t_i be a trip consisting of a sequence of bus stops and their destination school, and s_m be a school. In the figure, there are three, three, and four trips for school s_a, s_b, and s_c, respectively. Let $[s_m\_st, s_m\_et]$ be the time window for school m. All the students attending a school should arrive at the school within the bell time window. When two trips can be served by the same bus without violating the bus capacity constraint and the time window of their designated schools, they can be scheduled to the same bus. Fig. 2 shows a sample solution to the problem of Fig. 1. $\{t_2, t_6, t_9\}$, $\{t_1, t_4, t_8\}$, $\{t_3, t_7\}$, and $\{t_5, t_{10}\}$ are served by buses 1, 2, 3, and 4, respectively. It is assumed that $s_a\_et \leqslant s_b\_st$ and $s_b\_et \leqslant s_c\_st$.

In this paper, we first model the problem as a vehicle routing problem with time windows (VRPTW) by treating a trip as a virtual stop. However, the modeled VRPTW has a special characteristic due to the asymmetric nature of the original problem. Since the time windows of the schools are distant from each other and are not wide compared to the service times of the trips, it is assumed that when two trips can be served by the same bus, their sequence is fixed. Thus, when a trip can be followed by another trip by the same bus, the other sequence is not possible.

We show that a special case, wherein the start and end times of the trips are fixed and a homogeneous fleet of buses is given, can be modeled as an assignment problem and be easily solved. Then we propose an assignment problem based branch-and-bound approach for the general case with an assumption of a homogeneous fleet of buses. Finally, a heuristic algorithm is presented for the general case with the assumption of a heterogeneous fleet of buses. Computational experiments show the effectiveness of the proposed approaches.

The rest of this paper is organized as follows: a brief review of existing literature is presented in Section 2. Two mixed integer programming models and proposed algorithms for the problem are presented in Sections 3 and 4, respectively. Section 5 shows the computational results and concluding remarks follow in Section 6.

2. Literature review

The school bus routing problem includes three main sub problems, namely – bus stop selection, route (or trip) generation for each school, and bus scheduling problems. This section focuses on the review of the bus scheduling problem. Detailed descriptions of the general school bus routing problem and a survey of related literature can be found in Park and Kim (2010). Bus scheduling specifies the exact starting and ending time of each route and forms a chain of trips that can be executed successively by the same bus.

Gavish et al. (1978) solved a bus scheduling problem in which time tables (i.e., the starting and ending time) of trips are given.

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* Corresponding author. Tel.: +82 54 279 2371; fax: +82 54 279 2870.
E-mail address: bkim@postech.ac.kr (B.-I. Kim).

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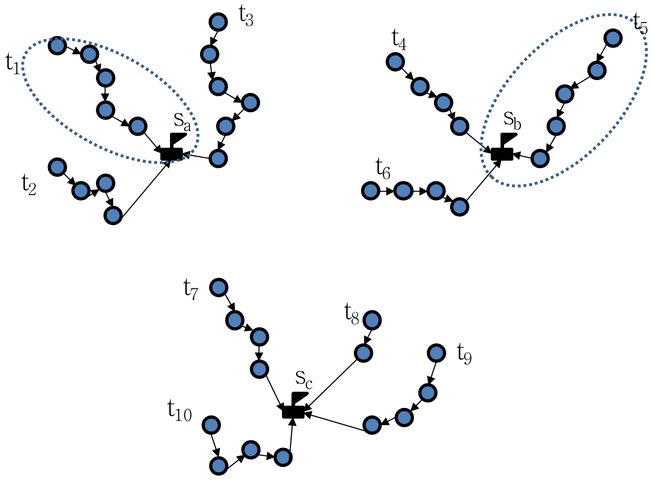

Fig. 1. A sample problem.

The problem was formulated as a transportation problem (TP). Orloff (1976), and Carraresi and Gallo (1984) handled the same bus scheduling problem but formulated it as an assignment problem (AP).

Löbel (1998a,b) considered the multiple depot vehicle scheduling problem (MDVSP) in public transit, which is a similar problem to the heterogeneous fleet school bus scheduling problem, formulated it as a multi-commodity flow problem, showed that it is NP-Complete even when the time tables of trips are given, and proposed a column generation approach as a solution method. While in the MDVSP, there are multiple depots, each vehicle belongs to one of the depots, and it should start from and end at its own depot, our problem does not have that constraint. Hadjar et al. (2006) proposed a branch-and-price-and-cut algorithm for the MDVSP. Pepin et al. (2009) compared five decomposition-technique-based heuristic methods and metaheuristics for the MDVSP. Desaulniers et al. (1998) developed an integer nonlinear mathematical model and a column generation embedded branch-and-bound algorithm for MDVRP with time windows (MDVRPTW). Hadjar and Soumis (2009) also proposed a branch-and-price algorithm with dynamic window reduction methods for the MDVRPTW.

Orloff (1976) proved that the homogeneous fleet school bus scheduling problem is NP-Complete when the arrival times of the trips are not fixed and the time windows of schools are given. He proposed a matching-based construction algorithm and a 3-opt improvement method for the problem.

Newton and Thomas (1974), Bodin (1975), and Bodin and Berman (1979) assumed that there are distinguished time periods, such that at the beginning of a period, buses would complete the duty of their previous period and would be located at their previous period destination schools. In each time period, each bus transports students for only a single school and makes one trip at most. The scheduling problem was solved by applying a series of TP solutions. However, their assumptions were overly simplified.

Swersey and Ballard (1984) proposed a nonlinear programming model for the school bus scheduling problem and its discretized approximation MIP models by dividing each school time window into a fixed number of arrival times. They solved the models using a linear relaxation and adjusted the solution manually when a solution had non-integer values.

Graham and Nuttle (1986) tested the algorithms of Orloff (1976), and Swersey and Ballard (1984). An AP-based algorithm was also tested, in which an initial solution was obtained by using AP formulation without considering time window constraints, and then its feasibility was handled manually. The study concluded that alternative heuristics that guarantee solution feasibility and short computation time are desired.

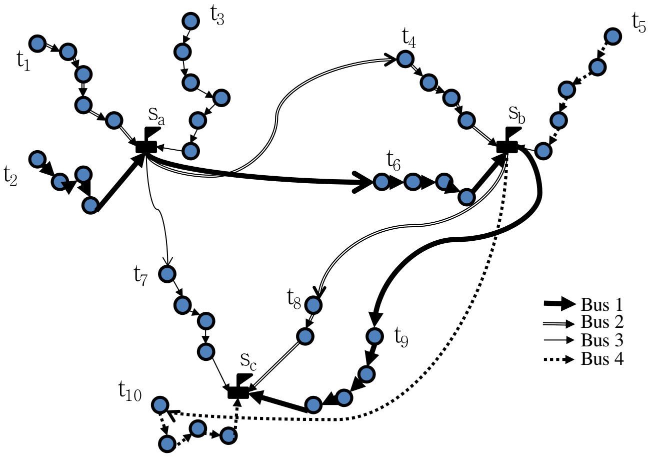

Fig. 2. A sample solution.

Braca et al. (1997) described the New York City school bus routing problem. While most literature solved the separate problem for each school, and determined the route schedule for each bus, they solved the whole problem of all schools in one stage using a randomized insertion-based heuristic.

Spada et al. (2005) considered a multiple school routing problem and proposed heuristic approaches. Schools are considered in increasing order of their starting times, and the routes for each school are built using a greedy method that considers the minimum vehicle capacity. Thereafter, the routes are merged if possible. Afterwards, the merged routes are improved by simulated annealing or a tabu search.

Desrosiers et al. (1981, 1986a) described a computerized school bus routing system consisting of student stop assignment, route generation, school start-time scheduling, and route scheduling modules. They assumed that the starting time of a school can be selected from a candidate list to reduce the number of buses required. With regard to the route scheduling module, which is a subject of this paper, they formulated and solved a series of TPs. Desrosiers et al. (1986b) developed three algorithms and tested them on eight school-transportation problems. They considered various settings, such as whether the problem occurred in the morning or in the afternoon, as well as the different interval lengths of time windows.

Fügenschuh (2009) also considered a school bus scheduling problem that permit the adjustment of school starting times and transshipment of students among trips. He formulated the problem as an integer programming model based on VRPTW, and the model was solved by a branch-and-cut approach with several pre-processing mechanisms and valid cuts. However, homogeneous fleet is assumed to be given and even a few small problems could not be optimally solved within the computation time limit (e.g., one hour) due to the complexity of the problem. Furthermore, the school start and trip start times are assumed fixed in their experiments for real-world problem sets.

In summary, the bus scheduling problem for homogeneous buses with fixed start and ending times can be solved by AP or TP. An AP formulation is also used for the problem in this paper. However, the bus scheduling problems with time windows are difficult, and there is no widely-used method yet. While some have attempted to solve the integrated routing and scheduling problem (Braca et al., 1997; Spada et al., 2005; Newton and Thomas, 1974; Bodin and Berman, 1979), some have separately focused on the scheduling problem (Swersey and Ballard, 1984; Desrosiers et al., 1981, 1986a,b; Fügenschuh, 2009). This paper is categorized in the latter case. Except Spada et al. (2005) and Fügenschuh (2009), all of the previous works studied the homogeneous vehicle problems. We will handle both homogeneous and heterogeneous fleet problems. To the