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Modeling age-based maintenance strategies with minimal repairs for systems subject to competing failure modes due to degradation and shocks

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ABSTRACT

This paper deals with maintenance strategies with minimal repairs for single-unit repairable systems which are subject to competing and dependent failures due to degradation and traumatic shocks. The main aims are to study different approaches for making a minimal repair decision (i.e., time-based or condition-based) which is a possible corrective maintenance action under the occurrence of shocks, and to show under a given situation which approach can lead to a greater saving in maintenance cost. Two age-based maintenance policies with age-based minimal repairs and degradation-based minimal repairs are modeled, and their performance is compared with a classical pure age-based replacement policy without minimal repairs. Numerical results show the cost saving of the maintenance policies and allow us to make some conclusions about their performance under different situations of system characteristic and maintenance costs. It is shown that carrying out minimal repairs is useful in many situations to improve the performance of maintenance operations. Moreover, the comparison of optimal maintenance costs incurred by both maintenance policies with minimal repairs allows us to justify the appropriate conditions of time-based minimal repair approach and condition-based minimal approach.

1. Introduction

With the growth in complexity of modern systems, maintenance strategies have become more important and play an essential role directly related to the competitiveness of organizations. Many mechanical and structural systems (e.g. cutting tool [1], hydraulic structure [2], airplane engine compressor blades [3], corroding pipelines [4], etc.) usually suffer an underlying degradation process which can cause random failures. Such degradation process can be modeled using stochastic processes. According to Lehmann [5], the stochastic-processes-based approach shows great flexibility in describing the failure-generating mechanisms and can give alternative time-to-failure-distributions defined by the degradation model.

In the literature, developments on maintenance models based on the degradation process have provided satisfactory results for the maintenance operation (e.g. see [6,7] for time-based maintenance model and [8–10] for condition-based maintenance model). But, considering a degradation process only, as in these papers, seems to be unsatisfactory for modeling dynamic systems that suffer a degradation with their operational age and that are subject to traumatic events or shocks which can lead to a sudden failure [11]. In these dynamic systems, the system is regarded as failed when its degradation reaches a critical threshold or when a catastrophic shock occurs although the degradation process has not reached the threshold. The models that describe these systems are called *Degradation-Threshold-Shock* (DTS) models. As far as we know, Lemoine and Wenocur were the first to analyze the DTS model [12]. Singpurwalla presents a comprehensive review of this class of models in [11]. Lehmann derived in [5] an expression of the survival function and the failure rate of the time to the system failure in the DTS model.

Most of the papers that deal with competing DTS models assume that the degradation process and the shock process are independent [5,13–15]. But, in many practical situations, the dependence between them is of importance and should not be neglected, as it leads to a competing risk model. Therefore, the present paper aims to expand the DTS model [5] by introducing the dependence between the shock process and the degradation process. The dependence between the shock process and the degradation process of the DTS model considered in this paper is described showing that the failure rate of the shocks depends on the degradation level of the system.

Based on such a degradation/failure model, this paper aims to develop a mathematical model to assess the benefit of carrying

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Acronyms

(τ, T) policy	age-based maintenance policy with minimal repairs depending on the system age
(A, T) policy	age-based maintenance policy with minimal repairs depending on the system degradation
PAR policy	pure (or classical) age-based replacement policy
RG	relative gain on the optimal maintenance cost rate
DTS	Degradation-Threshold-Shock

out minimal repairs as possible corrective maintenance actions under the occurrence of shocks. The basic concept of minimal repair was first introduced by Barlow and Hunter in [16], this has been extended in many ways, see e.g., [17–21]. The minimal repair is described by the fact that the repair action returns the system to an operational state but the system characteristics are the same as just before the failure. For a repairable system, carrying out minimal repairs is a natural approach, because it can keep the system working at a minimal cost. However, with a competing risk model under consideration as in this paper, performing a minimal repair requires more knowledge about the system state (e.g. failure type or degradation level) and cannot restore the system to as good as new, condition as in a system replacement, hence it can incur an unnecessary maintenance cost. So the question is whether to replace the system or to perform a minimal repair when the system fails, and under which conditions the minimal repair can delay the undertaking of system replacements at a more convenient time. We analyze whether carrying out a minimal repair should be based on the operational age of the system (time-based decision) or should be based on the degradation of the system (condition-based decision). To answer the above questions, we develop the maintenance cost models for two age-based policies: a policy implementing age-based minimal repairs at failure times, a policy implementing degradation-based minimal repairs at failure times. These policies are compared with a pure age-based replacement policy without minimal repairs in order to assess the value of the minimal repairs. Furthermore, comparing optimal maintenance costs incurred by both kinds of maintenance policy with minimal repairs allows us to give some conclusions about their performance under different situations of system characteristics and maintenance costs.

Hence, the main aims and contributions of the present study are:

1. Expanding DTS models introducing a dependence scheme between the shock process and the degradation process.
2. Developing the analytical cost models for the age-based maintenance policies which use different approaches to make a minimal repair decision.
3. Performing a quantitative comparison between the proposed maintenance models to assess the value of the minimal repair in the maintenance.
4. Providing the indicators for choosing the adequate maintenance policy according to different system characteristics.

The remainder of this paper is organized as follows. Section 2 is devoted to modeling the different competing failure modes of the system and to characterizing the associated failure times. The detailed description and formulation of the different age-based maintenance policies with and without minimal repairs are introduced in Section 3. Then in Section 4, we give the comparison of the relative gain in the optimal maintenance cost rate between the maintenance strategies under the different possible situations to assess the effect of resorting to minimal repairs and the value of investing in condition monitoring. Finally, the paper ends with some conclusions and directions for future works.

2. System degradation and failure modeling

The present paper considers a single-unit repairable system whose failures are due to the competing causes of degradation and shocks. The system is described by a so-called *Degradation-Threshold-Shock* (DTS) model [5]. In such a model, the degradation is modeled using a time-dependent stochastic process, and the system is regarded as failed when the degradation process reaches a critical threshold or when a shock occurs although the degradation process has not reached the threshold. As Singpurwalla advocated in [11], such a failure/degradation model can be seen as a combination and a more versatile-and hopefully realistic-extension of many classical failure models based either only on degradation or only on parametric lifetime distributions. More recently, Bocchetti et al. apply this model to describe the competing risks due to wear degradation and thermal cracking for the cylinder liners of a marine diesel engine [22]. In the following, the modeling of the different failure modes and the distributions of the associated hitting times are analyzed in detail, highlighting the dependence between the failure types.

2.1. Degradation-based failure

2.1.1. Degradation modeling

We consider a system deteriorating with use and age, and subject to a continuous accumulation of degradation. The degradation evolution of the system is modeled by a stochastic process. Let $X(t)$ be the accumulated degradation (or wear level) at time t. If no maintenance action is performed, the stochastic process $\{X(t), t \geqslant 0\}$ is continuous-time and monotonically increasing with $X(0) = 0$. The degradation is strictly increasing which means that the system worsens with time due to ageing and accumulated wear or damage.

We assume in this paper that $\{X(t), t \geqslant 0\}$ is a homogeneous gamma process as defined by Singpurwalla in [39] (by opposition to the gamma process defined by Berman in [36]). According to this definition, the gamma process is a monotone increasing stochastic process with continuous state space (often used to model a continuous degradation phenomena), completely defined by the law of the increments between two arbitrary instants (usually named increments of degradation) in the state space. This law is a gamma law. In the framework of continuous state degradation models, this definition of the gamma process is really mainly used [27]. The gamma processes were satisfactorily fitted to data of different gradual degradation phenomena such as erosion, corrosion, concrete creep, crack growth or wear of structural components [23–25]. Moreover, the existence of an explicit probability distribution function of the gamma process permits feasible mathematical developments. Therefore, since the initial proposal by Abdel-Hameed in [26], the gamma process has been analyzed for different maintenance applications by several authors (see [27] for a thorough review on the use of gamma process in maintenance modeling).