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A convex optimisation framework for the unequal-areas facility layout problem

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ABSTRACT

The unequal-areas facility layout problem is concerned with finding the optimal arrangement of a given number of non-overlapping indivisible departments with unequal area requirements within a facility. We present a convex-optimisation-based framework for efficiently finding competitive solutions for this problem. The framework is based on the combination of two mathematical programming models. The first model is a convex relaxation of the layout problem that establishes the relative position of the departments within the facility, and the second model uses semidefinite optimisation to determine the final layout. Aspect ratio constraints, frequently used in facility layout methods to restrict the occurrence of overly long and narrow departments in the computed layouts, are taken into account by both models. We present computational results showing that the proposed framework consistently produces competitive, and often improved, layouts for well-known large instances when compared with other approaches in the literature.

1. Introduction

The unequal-areas facility layout problem (FLP) is concerned with finding the optimal arrangement of a given number of non-overlapping indivisible departments with unequal area requirements within a facility. The objective of the FLP is to minimize the total expected cost of flows inside the facility, where the cost incurred for each pair of departments is taken as the rectilinear distance between the centroids of the departments times the projected flow between them. The projected flow may reflect transportation costs, the construction of a material-handling system, the costs of laying communication wiring, or even adjacency preferences among departments. The problem contains two sets of constraints: department area requirements and department location requirements (such as departments not overlapping, lying within the facility, and in some cases being fixed to a location, or being forbidden from specific regions). We assume that the facility and the departments are all rectangular. Since the height and width of the departments can vary, finding their optimal rectangular shapes is also part of the problem. The ratios height/width and width/height, called aspect ratios, also pose a challenge since departments with low aspect ratios are most practical in real-world applications, but this makes the problem harder. A solution to the FLP is a block layout that specifies the relative location and the dimensions of each department. Once a block layout has been achieved, a detailed layout can be designed which specifies department locations, aisle structures and input/output point locations [8,24,27,43].

A thorough survey of the facility-layout problem is given in [18], where the papers on facility layout are divided into three broad categories. The first is concerned with algorithms for tackling the FLP as defined above. The second category is concerned with extensions that take into account additional issues that arise in real-world applications, such as designing dynamic layouts by taking time-dependency issues into account, designing layouts under uncertainty conditions, and computing layouts that optimize two or more objectives simultaneously. The third category is concerned with specially structured instances of the problem, such as the layout of machines along a production line. In this paper, we shall focus exclusively on the block layout FLP.

The FLP as described above is a hard optimisation problem. In fact, even the restricted version where the shapes of the

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departments are all equal and fixed, and the optimisation is taken over a fixed finite set of possible department locations, is NP-hard. This restricted version is known as the quadratic assignment problem (QAP), see for example [37]. The largest QAP instances of the well-known Nugent set, those with 27, 28, and 30 departments, were solved to proven optimality using vast amounts of computational power and important improvements in mathematical programming algorithms [4].

Two types of approaches for finding provably optimal solutions for the FLP have been proposed in the literature. The first type are graph-theoretic approaches that assume that the desirability of locating each pair of facilities adjacent to each other is known. Initially, the area and shape of the departments are ignored, and each department is simply represented by a node in a graph. Adjacency relationships between departments can now be represented by arcs connecting the corresponding nodes in the graph. The objective is then to construct a graph that maximizes the weight on the adjacencies between nodes. We refer the reader to [17] for more details. The second type are mathematical programming formulations with objective functions based on an appropriately weighted sum of centroid-to-centroid distances between departments. Exact mixed integer programming formulations were proposed in [33,35], and nonlinear programming formulations are presented in some detail in Section 2 below. More recently, FLPs with up to eleven departments were solved to global optimality [39,12,11,32]. Thus, most of the approaches in the literature that tackle realistically sized problems are based on heuristics with no guarantee of optimality. These include genetic algorithms, tabu search, simulated annealing, fuzzy logic, and many others, see, e.g. [25,18,30,40].

The contribution of this paper is a two-stage convex-optimisation-based framework for efficiently finding competitive solutions for this problem. (Two-stage approaches for this problem using techniques different from ours are presented in [34,13].) The framework is based on the combination of two mathematical programming models. The first model is a convex relaxation of the layout problem that establishes the relative position of the departments within the facility, while the second model uses semidefinite optimisation to determine the final layout. Both models account for aspect ratio constraints, which are frequently used in facility layout methods to restrict the occurrence of overly long and narrow departments in the computed layouts. We present computational results showing that the proposed methodology consistently produces competitive, and often improved, layouts for well-known large instances when compared with other approaches in the literature.

This paper is structured as follows. In Section 2, the most recent nonlinear programming methods for the FLP are summarized. In Section 3, the proposed framework is motivated and derived. Computational results demonstrating the strength and potential of this framework are presented in Section 4. Finally, possible directions for future research are discussed in Section 5.

2. Previous nonlinear-programming-based methods

Throughout this paper we label the departments $i = 1,\ldots,N$, where N is the total number of departments. The position of each department i is expressed by the coordinates of its centre and is denoted by (x_i,y_i). It is assumed that the nonnegative costs c_{ij} per unit distance between departments i and j are given and are symmetric, i.e. $c_{ij} = c_{ji}$. We will approximate each department by a circle of radius r_i. The idea of using circular departments, or of approximating departments using circles, has been considered in several contexts (see for example [10,15,48] and the references therein).

We begin by describing the target distance methodology employed in [1,2]. Let each module i be represented by a circle of radius r_i, where r_i is proportional to $\sqrt{a_i}$, the square root of the area of module i. Following [1], we define the target distance for each pair of circles i, j as

$$t_{ij} = \alpha(r_i + r_j)^2,$$

where $\alpha > 0$ is a parameter. To prevent circles from overlapping, the target distance is enforced via the objective function by introducing a penalty term which acts as a repeller:

$$f\left(\frac{D_{ij}}{t_{ij}}\right),$$

where $f(z) = \frac{1}{z} - 1$ for $z > 0$, and $D_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2$. The objective function is thus given by

$$\sum_{1 \leqslant i < j \leqslant N} c_{ij}D_{ij} + f\left(\frac{D_{ij}}{t_{ij}}\right).$$

The interpretation here is that the first term is an attractor that makes the two circles move closer together and pulls them towards a layout where $D_{ij} = 0$, while the second term is a repeller that prevents the circles from overlapping. Indeed, if $D_{ij} \geqslant t_{ij}$ then there is no overlap between circles and the repeller term is zero or slightly negative, while the attractor in the objective function applies an attractive force to the two circles. On the other hand, if $D_{ij} < t_{ij}$ then the repeller term is positive, and it approaches positive infinity as D_{ij} tends to zero, preventing the circles from overlapping completely.

In summary, the model aims to ensure that $\frac{D_{ij}}{t_{ij}} = 1$ at optimality, so choosing $\alpha < 1$ sets a target value t_{ij} that allows some overlap of the areas of the respective circles, which means that a relaxed version of the non-overlap requirement of the circles is enforced. In practice, by properly adjusting the value of α, we achieve a reasonable separation between all pairs of circles. The complete attractor-repeller (AR) model as given in [1] is:

$$\min_{(x_i,y_j),w_F,h_F} \quad \sum_{1 \leqslant i < j \leqslant N} c_{ij}D_{ij} + f\left(\frac{D_{ij}}{t_{ij}}\right), \tag{1}$$

$$\text{s.t.} \quad x_i + r_i \leqslant \frac{1}{2}w_F \quad \text{and } r_i - x_i \leqslant \frac{1}{2}w_F, \quad \text{for } i = 1,\ldots,N, \tag{2}$$

$$y_i + r_i \leqslant \frac{1}{2}h_F, \quad \text{and } r_i - y_i \leqslant \frac{1}{2}h_F, \quad \text{for } i = 1,\ldots,N, \tag{3}$$

$$w_F^{low} \leqslant w_F \leqslant w_F^{up}, \tag{4}$$

$$h_F^{low} \leqslant h_F \leqslant h_F^{up}, \tag{5}$$

where (x_i,y_i) are the coordinates of the centre of circle i as previously defined; w_F, h_F are the width and height of the facility; and w_F^{low}, w_F^{up}, h_F^{low}, and h_F^{up} are the lower and upper bounds of the width and the height of the facility, respectively. The first two sets of constraints require that all the circles be entirely contained within the facility, and the remaining two pairs of inequalities bound the width and height of the facility. (Note that the geometric centre of the facility outline is at the origin of the $x - y$ plane.)

An important drawback of model (1)–(5) is that the objective function is not convex, and hence the overall model is not convex. By modifying it so as to obtain a convex problem, we expect to obtain a relaxation that captures better global information about the problem. Also, note that there is no force between i and j if $D_{ij}^2 = t_{ij}/c_{ij}$. For these reasons, the analysis in [1,2] motivates the definition of the following generalized target distance T_{ij}:

$$T_{ij} := \sqrt{\frac{t_{ij}}{c_{ij} + \varepsilon}},$$

where $\varepsilon > 0$ is a sufficiently small number such that if $D_{ij} \approx T_{ij}$ then $D_{ij} \approx \sqrt{t_{ij}/c_{ij}}$. This target distance takes both the relative size of the