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Two shock and wear systems under repair standing a finite number of shocks

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ABSTRACT

A shock and wear system standing a finite number of shocks and subject to two types of repairs is considered. The failure of the system can be due to wear or to a fatal shock. Associated to these failures there are two repair types: normal and severe. Repairs are as good as new. The shocks arrive following a Markovian arrival process, and the lifetime of the system follows a continuous phase-type distribution. The repair times follow different continuous phase-type distributions, depending on the type of failure. Under these assumptions, two systems are studied, depending on the finite number of shocks that the system can stand before a fatal failure that can be random or fixed. In the first case, the number of shocks is governed by a discrete phase-type distribution. After a finite (random or fixed) number of non-fatal shocks the system is repaired (severe repair). The repair due to wear is a normal repair. For these systems, general Markov models are constructed and the following elements are studied: the stationary probability vector; the transient rate of occurrence of failures; the renewal process associated to the repairs, including the distribution of the period between replacements and the number of non-fatal shocks in this period. Special cases of the model with random number of shocks are presented. An application illustrating the numerical calculations is given. The systems are studied in such a way that several particular cases can be deduced from the general ones straightaway. We apply the matrix-analytic methods for studying these models showing their versatility.

1. Introduction

Shock and wear systems have been frequently studied in the literature, since they introduce environment conditions and wear in the internal structure that modify the lifetime of the systems. These systems can fail due to these two causes: external and internal. The environment in which the systems operate is incorporated to the study by means of a model of arrival of shocks. In the literature, several models have been considered for the arrival processes. An usual maintenance operation of the systems is the repair. The replacements are due to the arrival of non-repairable failures. When repairs are as good as new (perfect repair), every time that the system completes a repair can be considered a replacement. All these operations together complicate the study of the systems. In the present paper we consider the matrix-analytic methods for studying shock and wear models under repair.

The shock and wear systems have been studied in the literature extensively under different methodologies. On the other hand, repair and replacement are operations usually applied in reliability. The following list is a selection of papers related to shocks, repair and replacements. In [1] a system subject to shocks arriving following a non-homogeneous Poisson process and producing two types of failures is studied, determining the optimal costs of the replacements. In [2] a system subject to two types of failures producing minimal repair and replacement is studied, calculating the expected cost rate of the maintenance and discussing the minimization of this quantity. In [3] the availability of a system maintained through several imperfect repairs before a replacement or a perfect repair is calculated, using the Fourier transformation technique. In [4] the optimal policy of age-replacement of a system subject to shocks under minimal repair is calculated. In [5] an optimal number of minimal repairs before replacement based on a cumulative repair-cost is studied. All these papers do not use matrix-analytic methods. The following list of papers is restricted to those using these methods, characterized by the inclusion of phase-type distributions and Markovian arrival processes. Phase-type distributions have been used in the study of reliability systems in [6] for studying a system under two types of failures under policy N. Shock and wear models are studied under matrix-analytic methods for the first time in [7], and the survival function of the system is calculated. Preventive maintenance for inspected systems under shocks with cumulative damage is studied in [8]. In [9] a shock and wear model with operational and repair times

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governed by phase-type distributions and with interarrival times depending on the number of cumulated shocks interarrival times is studied, and the survival function calculated. The availability of inspected systems subject to shocks by using a matrix algorithmic approach is calculated in [10]. In [11] the optimal structured replacement policies for a periodically inspected system subject to shocks arriving according to a Markov-modulated Poisson process is established. A system subject to wear governed by phase-type distribution and external failures arriving following a Markovian arrival process with two types of failures is studied in [12], and different performance measures are calculated. In [13] a maintenance model with failures, minimal repairs, and inspections following a Markovian arrival process is studied. The replacement policy in shock and wear models considering a Markovian arrival process as a model of arrival of shocks under minimal repair are studied in [14]. In [15] a shock model under a Markovian arrival process with two types of failures, fatal and non-fatal, and with no limitation of the number of shocks is studied, calculating the counting process of the number of shocks.

We present a shock and wear system subject to shocks arriving following a Markovian arrival process. The lifetime of the system follows a phase-type distribution. The number of shocks that the system can stand is finite. The system may fail because of wear according to the lifetime or because a critical number of shocks is attained. Given that there are two types of failures, we add the option of two types of repair. If the failure is due to wear, the system is repaired, this process requires a random time governed by a phase-type distribution and it can be considered as normal repair. If the failure is due to the arrival of a critical number of shocks, the repair time follows a phase-type distribution different from the normal repair, it can be considered severe repair. Both repairs are as good as new. Under these conditions, two systems are studied depending on the number of shocks causing the failure of the system can be random or fixed. The Markov process governing the systems are constructed and special cases of the general systems can be deduced straightaway. For these systems the main performance measures are calculated and the renewal process governing the replacements of the systems associated to the repairs are studied.

The paper contributes to the study of reliability systems in several ways. The interarrival times in a Markovian arrival process are not independent, and the phase-type distributions are a class dense in the sense of distributions in the family of distributions defined on the positive real half-line. These assumptions imply that the shock and wear models we consider extend previous papers by considering a more general arrival process and approximating any lifetime and repair distribution by a phase-type one. Two types of repair are introduced, normal and severe, depending on the type of failure; in this way, we introduce a versatile scheme for repairing. The renewal process associated to the perfect repairs is studied, calculating the distribution of the time between consecutive repairs and the number of repairs in a period of time. The system standing a fixed number of shocks is not deduced straightaway from the system with a random critical number of shocks, as we will see, and therefore we study them separately. In the study of the general models are included indicators representing the inclusion or not of the different types of repair. This allows us to study directly particular cases of the systems with one or non-repair. We study in detail the general models in which two types of repair are included, and indicate how to study the others.

The paper is organized as follows. In Section 2 the general model is presented and some performance measures are calculated. In Section 3 the number of shocks is fixed, and the corresponding system is studied. In Section 4 the model with two types of repair is studied. In Section 5 are presented special cases of the general model. In Section 6 a numerical application is performed. In Appendices A, B, C matrices for the models with two types of repair are given.

1.1. Definitions

The phase-type distributions, the Markovian arrival process, and the operations of Kronecker play an important role in the present paper. We define them formally.

Definition 1. The distribution $H(\cdot)$ on $[0,\infty]$ is a continuous *phase-type distribution*(PH-distribution), if it is the distribution of the time until absorption in a Markov process with a finite number of transient states and one absorbent one. Denoting the matrix of rates among the transient states by T, the initial probability row vector restricted to the transient states by α, and being e the column vector of 1's with the same order of α, the distribution $H(\cdot)$ is given by

$$H(x) = 1 - \alpha \exp(Tx)e, \quad x \geqslant 0.$$

It will be denoted that $H(\cdot)$ follows a $PH(\alpha,T)$ distribution. The absorption vector T^0 satisfies $-Te = T^0 \geqslant 0$. It is said that the distribution has representation (α,T). The order of the distribution is the number of transient states.

Definition 2. A discrete phase-type is defined similarly to the continuous case for a Markov chain. Denoting the probability matrix among the transient states by S, the initial probability row vector restricted to the transient states by β, and being S^0 the column vector satisfying $Se + S^0 = e$ the column vector of 1's with the same order of β, the probabilities $\{p_k\}$ are given by

$$p_k = \beta S^{k-1} S^0, \quad k \geqslant 1.$$

It will be written that $\{p_k\}$ follows a $PH_d(\beta,S)$. The order of the distribution is the number of transient states.

Definition 3. Let $\mathbf{D}$ be an irreducible infinitesimal generator of dimension m. We consider a sequence of matrices $\mathbf{D}_k$, $k \geqslant 1$, of dimension m, non-negative and the matrix $\mathbf{D}_0$ has non-negative off-diagonal entries. The diagonal entries of $\mathbf{D}_0$ are strictly negative and it is non-singular. The sum of the matrices $\mathbf{D}_k$, $k \geqslant 0$, is the given matrix $\mathbf{D}$. Consider an m-state Markov renewal process $\{(J_n,X_n), n \geqslant 0\}$ in which each transition epoch has an associated positive number that indicates the type of arrivals $\mathbf{L}_n$. The random variables J_n, X_n are the states and the sojourn times in states, respectively. The transition probability matrix with $(j,j') -$ entries $P\{J_n = j', \mathbf{L}_n = k, X_n \leqslant x | J_{n-1} = j\}$ is given by

$$\int_0^x \exp(\mathbf{D}_0 u) du \mathbf{D}_k, \quad \text{for } k \geqslant 1, \quad x \geqslant 0.$$

The Markovian arrival process (MAP) is defined as the Markov renewal process and a transition probability matrix of the stated particular form.

Definition 4. If A and B are rectangular matrices of orders $m_1 \times m_2$*and* $n_1 \times n_2$, respectively, their Kronecker product $A \otimes B$ is the matrix of order $m_1 n_1 \times m_2 n_2$, written in compact form as $(a_{ij}B)$. The Kronecker sum of the square matrices C and D of orders p and q, respectively, is defined by $C \oplus D = C \otimes I_q + I_p \otimes D$, where I_k denotes the identity matrix of order k.

For more details about these operations, results, examples and applications see [16], [17], and for phase-type distributions and Markovian arrival process see [18], [19].