European Journal of Operational Research 214 (2011) 348–357

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Stochastics and Statistics

From deterministic to stochastic surrender risk models: Impact of correlation crises on economic capital

Stéphane Loisel, Xavier Milhaud *

Université de Lyon, Université Claude Bernard Lyon 1, Institut de Science Financiére et d'Assurances, 50 Avenue Tony Garnier, F-69007 Lyon, France

ARTICLE INFO

Article history:
Received 8 July 2010
Accepted 29 April 2011
Available online 7 May 2011

Keywords:
Risk management
Applied probability
Life insurance
Surrender risk
Correlation risk

ABSTRACT

In this paper we raise the matter of considering a stochastic model of the surrender rate instead of the classical S-shaped deterministic curve (in function of the spread), still used in almost all insurance companies. For extreme scenarios, due to the lack of data, it could be tempting to assume that surrenders are conditionally independent with respect to a S-curve disturbance. However, we explain why this conditional independence between policyholders decisions, which has the advantage to be the simplest assumption, looks particularly maladaptive when the spread increases. Indeed the correlation between policyholders decisions is most likely to increase in this situation. We suggest and develop a simple model which integrates those phenomena. With stochastic orders it is possible to compare it to the conditional independence approach qualitatively. In a partially internal Solvency II model, we quantify the impact of the correlation phenomenon on a real life portfolio for a global risk management strategy.

Surrender risk represents one of the main dangers faced by a life insurance institution. It corresponds to the risk that many policyholders surrender their contract earlier than expected and choose to reinvest their money in another product or in some project. Because fees are charged throughout the duration of the contract, the insurer may not have enough time to charge the fees in the case of early surrenders. Massive early surrenders might also cause important liquidity issues and of course a loss of market share. Many policyholders surrender their life insurance contract every year, mainly to finance a project (building a new house, purchasing a new car, ...) or because the tax incentive delay (8 years in France) or the penalty relief delay has been reached. Insurers are used to forecasting lapse rates, which may be explained by different factors like age, wealth and so on. In Solvency II, internal or partially internal models are being developed by many companies (see Devineau and Loisel, 2009 for a description and comparison of some internal models with the standard formula). They have to go from a deterministic model, often based on a so-called S-shaped lapse rate curve to a stochastic model. The S-shaped curve corresponds to the lapse rate expressed as a function of the difference Δr between the interest rate given by the contract and the one that the policyholder could obtain somewhere else in the market. The idea that practitioners have followed is that even if Δr is very small, some policyholders are going to surrender their contract for tax reasons or to fund a personal project, that the lapse rate is increasing in Δr, and that even if Δr is very large, some policyholders are going to stay in the portfolio because they do not really pay attention to the market evolution. The problem with this S-shaped curve is that one has not observed policyholders' behavior in the extreme situation in which Δr is very large.

To build a stochastic model, given this lack of information, one must more rely on thought experiments than on statistical data (which simply does not exist). It could be tempting for internal model designers to use a Gaussian distribution around the value of the lapse rate in the S-shaped, deterministic curve to describe stochastic surrender risk. In this paper, we explain why it may be preferable to use a bi-modal distribution, due to the likely change in the correlation between policyholders' decisions in extreme scenarios. This change of correlation in extreme situation, called correlation crisis in Biard et al. (2008), Loisel (2010) and Loisel (2008) prevents us from applying the classical Gaussian approximation based on the central limit theorem. This theorem holds when decisions of policyholders are independent. Here, this would be the case only given a certain factor that would incorporate the level of information of policyholders and the reputation of the company and of the insurance sector. This factor should be a key element in the internal model to understand the correlation of surrender risks with other risks like market risk or default risk. Correlation crises have followed the sub-prime crisis in both stock markets and credit derivatives market: in many cases, correlation increases in adverse scenarios. For surrender risk, it is likely that an extreme

* Corresponding author. Address: Axa Global Life, 40 rue du Colisée, 75008 Paris. Tel.: +33 156 437 987.

E-mail addresses: stephane.loisel@univ-lyon1.fr (S. Loisel), xavier.milhaud@gmail.com (X. Milhaud).

0377-2217/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2011.04.038

situation in interest rates markets would lead either to massive surrenders, or to almost normal lapse rates, depending on political declarations and on other factors: for example, one of the first things that leaders of developed countries said at the beginning of the last crisis was: *We guarantee bank deposits and classical savings products*. This leads to anticipate policyholders' behavior more like a $0-1$ law than according to a bell-shaped unimodal distribution. In this paper, we propose a basic model that takes into account correlation crises: as Δr increases, correlation between policyholders' decisions increases, and one goes (continuously) from a bell-shaped distribution in the classical regime to a bi-modal situation when Δr is large. The model is proposed in Section 1 and interpreted in Section 2. In Section 3, we explain how to compute surrender rate distributions, with closed formulas and with simulations. In Section 4, we make use of stochastic orderings in order to study the impact of correlation on the surrender rate distribution from a qualitative point of view. In Section 5, we quantify this impact on a real-life portfolio for a global risk management strategy based on a Solvency II partial internal model.

1. The model

Assume that when Δr is zero, policyholders behave independently with average lapse rate $\mu(0)$, whereas when Δr is very large (15%, say), the average lapse rate is $1-\epsilon$ with ϵ very small, and correlation between individual decisions is $1-\eta$, with η very small. The following model captures these simple features: let I_k be the random variable that takes value 1 if the kth policyholder surrenders her contract, and 0 otherwise. Assume that

$$I_k = J_k I_0 + (1-J_k) I_k^{\perp},$$

where J_k corresponds to the indicator that the kth policyholder follows the market consensus (copycat behavior). The random variable J_k follows a Bernoulli distribution whose parameter p_0 is increasing in Δr, and $I_0, I_1^{\perp}, I_2^{\perp}\ldots$, are independent, identically distributed random variables, whose parameter p is also increasing in Δr. This means that the surrender probability increases with Δr, and that the correlation (Kendall's τ or Spearman ρ) between I_k and I_l (for $k \neq l$) is equal to $P(J_k = 1|\Delta r = x)$ when $\Delta r = x$, and that in general (without conditioning) the correlation between I_k and I_l (for $k \neq l$) is equal to

$$\int_0^{+\infty} P(J_k = 1|\Delta r = x)\, dF_{\Delta r}(x).$$

This is because given that $\Delta r = x$, I_k and I_l (for $k \neq l$) admit a Mardia copula (linear sum of the independent copula and of Fréchet upper bound).[1] For a portfolio of 20000 policyholders, the Gaussian assumption is not too bad for the case where $\Delta r = 0$. We show here with realistic values of the S-shaped curve how this bell-shaped curve progressively evolves as Δr increases and at some point $\Delta r = x_0$ becomes bi-modal. McNeil et al. (2005) perfectly illustrates the problem of correlation risk and its consequences on tail distribution in a general context.

2. Interpretation of the model

The S-shaped curve of the surrender rate in function of Δr on Fig. 1 shows that the less attractive the contract is, the more the policyholder tends to surrender it. Obviously the surrender rate average is quite low in a classical economical regime (Region 1, low Δr on Fig. 1), but is significantly increasing as Δr increases. Indeed when interest rates rise, equilibrium premiums decrease and

[1] Here the copula of I_k and I_l (for $k \neq l$) is not unique as their distributions are not continuous.

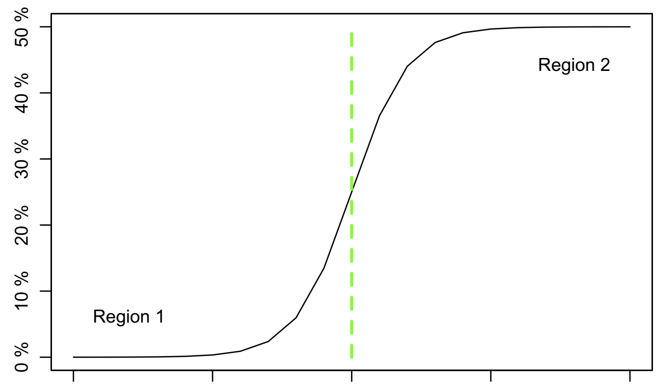

Fig. 1. Surrender rate versus Δr.

a newly acquired contract probably provides the same coverage at a lower price: the investor acts as the opportunity to exploit higher yields available on the market. On the contrary, if the interest rates drop then the guaranteed credited rate of the contract may be (when it is possible) lowered by the insurer (for financial reasons or to stimulate the policyholder to surrender).

By consequence, Region 1 in Fig. 1 illustrates the case corresponding to independent decisions of policyholders (here the correlation tends to 0) whereas Region 2 corresponds to much more correlated behaviors (correlation tends to 1 in this situation) because of a crisis for instance. The underlying idea of the paper is that as long as the economy remains in "good health", the correlation between policyholders is quasi nonexistent and thus the surrender rate (independent individual decisions) can be modeled thanks to the Gaussian distribution whose mean and standard deviation are those observed. Indeed the suitable distribution in Region 1 is the classical Normal distribution represented in Fig. 2.

On the contrary the sharp rise of the surrender rate at some level Δr in Fig. 1, followed by a flat plateau which is the maximum reachable surrender rate (this bound is often suggested by an expert since we consider that we have never observed it), reflects that economical conditions are deteriorating. The crucial point is to realize that in such a situation the assumption of independent behaviors can become strongly erroneous: the correlation between policyholders' decisions makes the surrender rate distribution change. This is the consequence of two different behaviors or scenarios, either almost all policyholders surrender their contract or they do not. The more suitable distribution to explain it is the so-called *Bi-modal* distribution illustrated in Fig. 2. The main difference with the Gaussian model is that the average surrender rate results from two peaks of the density.

Note that irrational behavior of policyholders could also lead to correlation crises between their decisions even if Δr is small. We shall see that this situation is the one that has the strongest impact on economic capital needs. Irrational behavior must be understood here as atypical with respect to the historical records of the insurance company, following some rumor or some recommendation from journalists or brokers. From a financial perspective, irrational behavior corresponds to the one of policyholders who do not surrender their contract even if it would pay to surrender it. As life insurance contracts feature more and more complex embedded options or guarantees, and as tax incentives are at stake, it might be difficult for the policyholder to use them optimally. However, it can be noticed in the US life insurance market (in which many variable annuities are present) that policyholders seem to become more and more rational. This uncertainty on future policyholder's rationality is somehow partly captured by our correlation crisis model.