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Dynamic lot sizing for multiple products with a new joint replenishment model

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ABSTRACT

This paper studies a new multi-product dynamic lot sizing problem, where the inventories of all products are replenished jointly with the same quantity whenever a production occurs. Such problems may occur in poultry and some chemical industries. We first introduce the general problem that allows for demand rejection with lost sales cost, and prove that the problem is NP-hard. Then we study a special case where all demands have to be satisfied immediately, and show that it can be solved in polynomial time. Finally, we develop two heuristic algorithms for the general problem. Through computational experiments, we demonstrate the effectiveness of the heuristics and investigate some insights related to the decision of lost sales.

1. Introduction

We study a production planning problem for multiple products. The problem is novel in that the inventories of all products are replenished jointly whenever a common batch production occurs, and the output of any production batch always produces each individual product along a fixed ratio. For example, suppose that there are three products A, B, and C with ratio 2:3:4. Then given a production quantity x, we will have $2x$ units of A, $3x$ units of B, and $4x$ units of C. The production can also be regarded as a process of making a virtual "composite" product which will then be immediately decomposed into many types of specific products following a given ratio. In the mean time, the demand for each product may be different from each other and, in particular, inconsistent with the production output ratio. The production planning problem is to decide the production quantity of the "composite" product at any time in a planning horizon. The challenge is how to determine a single production quantity that has to simultaneously take care of the heterogenous demands of multiple products. To simplify the presentation, we will assume that each production will generate the same quantity of output for every individual product. This, however, does not lose any generality because we can always redefine the "unit" of each individual product.

The problem is directly motivated by the poultry industry. To better satisfy customers' demands and pursue higher profit margins, a poultry company usually sells different parts of the chicken separately, for example, breast meat, thighs, wings, and even necks and fingers. However, as we all know, no matter how much chicken is killed, these parts will always be produced with a fixed ratio. Thus the production planning for all these parts is correlated by the common production process, and the decision is essentially the quantity of chicken to be killed. Similar situation can also be observed in the chemical industry where the production of some products will generate some other by-products that can also be sold, and the production quantity of each by-product is linearly proportional to the quantity of the main product.

We will study the production planning problem for such multiple products under the dynamic lot sizing model. Specifically, there are N types of products, the planning horizon contains T time periods, and the demand for each product in each time period is given. In each period, we have a production setup cost, production variable cost, product-specific inventory holding cost, and lost sales cost to be considered as the tradeoffs in the decision.

Besides the joint replenishment with a fixed production output ratio for multiple products, another important issue emphasized in our dynamic lot sizing model is to allow for lost sales or order rejection. In general, when supply is below demand in one period, the unsatisfied demand can be handled by two different ways, to be backlogged to the next period, or to be lost immediately in the current period. Considering lost sales has not been a main stream factor in dynamic lot sizing models, at least compared with backlogging. However, in our problem, allowing for lost sales is especially relevant due to the fact that each product usually has a distinct economic value. There may be a few major products that have high profit margins, and also other by-products with low profit margins. If, at any particular time, there is a demand burst for any by-product, it may be wise to reject some such demand. Therefore, we will include the option of lost sales in our model.

In this paper, we focus on solving the dynamic lot sizing problem for such multiple products. Our contribution can be summarized as follows.

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First, we consider a simple case where the demands of all products have to be satisfied without any lost sales. We show that the problem has an optimal structural property similar to the traditional single-product dynamic lot sizing model, and can be solved in $O(NT^2)$ time by a dynamic programming algorithm.

Second, we consider the more general case, where lost sales of some demands are allowed. We show that the problem will become NP-hard even if there are only two products, and hence we need to develop heuristic algorithms. Intuitively, there will be two approaches to the heuristics, considering products one by one, and considering all products simultaneously. We develop different heuristic algorithms following these two approaches.

Third, we conduct computational experiments to test the performance of the algorithms and study managerial insights. We find that the heuristic of considering products one by one can find good solutions under certain conditions, and the heuristic of considering all products simultaneously can always find solutions very close to the optimal ones. Comparing different products, we find that lost sales may occur more often to the products with less economic values.

The rest of this paper is organized as follows. In Sections 2, we have a brief literature review. In Section 3, we formally present and formulate the problem. In Section 4, we study the case without lost sales, and in Section 5, we study the case with lost sales. We present computational results in Section 6 and conclude the paper in Section 7.

2. Literature review

The dynamic lot sizing problem (DLS), as one of the fundamental OR/OM models, has received enormous attention in the past few decades. The work is too extensive to review here. Our work belongs to multi-item DLS problems. In such problems, there are multiple products to be produced in each production lot, where these products share either a common setup cost (e.g., Kao, 1979; Federgruen and Tzur, 1994) or a limited production capacity (e.g., Gilbert, 2000; Boctor and Poulin, 1994; Bruggemann and Jahnke, 2000; Ferreira et al., 2009). However, in all existing works on multi-item DLS models, there is no constraint on the production quantity of each individual product; a solution is feasible as long as the total production quantity is within the capacity, if any. This is quite different from our problem that has a fixed production output ratio for the individual products.

Our work is also related to DLS with lost sales, where lost sale refers to the case in which some demand in one period will be lost immediately with certain lost sales cost. This can also be regarded as the supplier rejecting some demand. In the literature, single-item DLS models with lost sales have been addressed. Aksen et al. (2003) developed a dynamic programming algorithm to solve the problem, and Aksen (2007) further generalized the model to consider the impact of lost sales on the future demand. Multi-item DLS with lost sales was addressed recently in Absi and Kedad-Sidhoum (2008), but as we explained, our multi-product problem is a new model compared with the conventional multiple-item DLS.

Beyond the research on DLS, our problem can be regarded as a special case of disassembly scheduling (Lee et al., 2002; Kim et al., 2007) which studies how to decompose a product into multiple subproducts. In general, the research on disassembly scheduling assumes a multi-level subproduct disassembly structure, and disassembling each subproduct requires certain time duration. Consequently, that leads to complicated mathematical formulations and solution approaches. Our problem is a single-level disassembly problem, the special structure of which enables us to develop more efficient specialized algorithms.

A very similar production control model, known as co-production in the literature, has been studied in the semi-conductor industry, for example, Bitran and Gilbert (1994), Gerchak et al. (1996) and Öner and Bilgiç (2008). In co-production problems, each production batch also generates multiple products, the same as our problem. However, there are two main differences. First, in co-production, the output percentage of each product is a random variable, but in our problem the output percentage is deterministic. Second, which is more important, the multiple products in co-production often refer to different grades of one generic product, and consequently, exploiting the opportunity of demand substitution among different products is a major decision. However, demand substitution is not allowed in our problem.

3. Problem formulation and analysis

Suppose that there are multiple types of products sharing the same production process, where the output of the products in any production batch follows a given fixed ratio. Without loss of generality, we assume that each batch will generate the same number of "units" of all products. The assumption is possible by properly scaling the demands of all products.

Suppose that the planning horizon contains multiple discrete time periods, where each period has a demand of each product and a known cost structure. The decision is the production quantity in each period. Note that each production will increase the inventory levels of all products by the same quantity due to the assumption of scaled demand.

We have the following notation to define the input of the problem:

N: number of product types;
T: number of time periods;
d_{it}: demand for product i in period t;
h_{it}: holding cost for one unit of product i in period t;
l_{it}: lost sales cost for one unit of product i in period t;
A_t: production setup cost in period t;
p_t: unit production cost in period t;
D_{it}: cumulative demand for product i from period 1 to period t, i.e., $D_{it} = \sum_{j=1}^{t} d_{ij}$;
$\overline{D}_t$: maximum cumulative demand among all products from period 1 to period t, i.e., $\overline{D}_t = \max_{1\leqslant i\leqslant N}\{D_{it}\}$.

The problem, denoted by **Problem P**, is to serve the demands, by allowing for lost sales, at the minimum total cost. Specifically, we have the following decision variables:

x_t: the production quantity in period t;
z_t: a binary indicator of the production decision in period t, i.e., $z_t = 1$ if and only if $x_t > 0$;
y_{it}: the quantity of demand rejected for production i in period t;
I_{it}: the inventory level of product i in period t, where we assume $I_{i,0} = 0$ for $i = 1,\ldots,N$.

We then have the following mixed integer linear programming (MIP) formulation to define the problem.

Problem P:

$$\text{Minimize} \quad \sum_{t=1}^{T}(A_t z_t + p_t x_t) + \sum_{i=1}^{N}\sum_{t=1}^{T}(h_{it}I_{it} + l_{it}y_{it}) \tag{1}$$

$$\text{subject to} \quad x_t \leqslant M z_t, \quad t = 1,\ldots,T, \tag{2}$$

$$I_{i,t-1} + x_t + y_{it} = I_{it} + d_{it}, \quad i = 1,\ldots,N, \quad t = 1,\ldots,T, \tag{3}$$

$$y_{it} \leqslant d_{it}, \quad i = 1,\ldots,N, \quad t = 1,\ldots,T, \tag{4}$$

$$x_t \geqslant 0, \quad y_{it} \geqslant 0, \quad I_{it} \geqslant 0, \quad z_t \in \{0,1\}, \quad i = 1,\ldots,N, \quad t = 1,\ldots,T. \tag{5}$$