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GRASP heuristic with path-relinking for the multi-plant capacitated lot sizing problem ☆

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ABSTRACT

This paper addresses the independent multi-plant, multi-period, and multi-item capacitated lot sizing problem where transfers between the plants are allowed. This is an NP-hard combinatorial optimization problem and few solution methods have been proposed to solve it. We develop a GRASP (Greedy Randomized Adaptive Search Procedure) heuristic as well as a path-relinking intensification procedure to find cost-effective solutions for this problem. In addition, the proposed heuristics is used to solve some instances of the capacitated lot sizing problem with parallel machines. The results of the computational tests show that the proposed heuristics outperform other heuristics previously described in the literature. The results are confirmed by statistical tests.

1. Introduction

The capacitated lot sizing problem (CLSP) is a combinatorial optimization problem whose objective is to find a production plan that minimizes production, setup, and inventory costs, and meets without delay the demands of items in the periods in the planning horizon. According to Karimi et al. (2003), the CLSP is one of the most important and difficult problems in tactical production planning. For the case in which setup times are considered, the problem to find a feasible solution is NP-complete (Maes et al., 1991). This problem has been studied widely (Trigeiro et al., 1989; Lozano et al., 1991; Diaby et al., 1992a,b; Armentano et al., 1999). Moreover, numerous surveys have been published (Bahl et al., 1987; Kuik et al., 1994; Wolsey, 1995; Karimi et al., 2003).

According to Bahl et al. (1987), one can classify lot sizing problems as single stage (with one planning stage) or multi-stage (with several planning stages). A system has a single stage when the items to be produced are independent, i.e., one item does not depend on the other to be produced. On the other hand, a multi-stage system is characterized by the fact that production of each item generates dependent demand for its components, whose production or purchase should also be planned.

The CLSP with parallel machines consists of a limited number of machines (or production lines) where any machine can produce the same items in an environment composed of a single stage and one plant. The machines can have different production and setup costs, and can as well be capacitated. This problem has been studied by Lasdon and Terjung (1971), Carreno (1990), and Toledo and Armentano (2006).

In this paper, we address the single stage, multi-plant, multi-item, and multi-period capacitated lot sizing problem (MPCLSP). The problem considers transfer costs among plants and individual per period–plant demands. These transfer costs are incurred because we allow a plant to produce items for another plant. Likewise, we allow storage of items in plants distinct from the one in which the item is produced and/or is demanded. Since customers only pay for transportation from the nearest plant to the delivery location, the eventual additional transfer costs must be accounted for. Since the problem to find a feasible solution to the single plant capacitated lot sizing problem with setup time is NP-complete, so is its multi-plant variant. As we show later, exact methods encounter difficulties to solve instances of moderate size. Therefore, the use of heuristics as solution methods for this problem is justified. Some applications of these problems can be found in a wide range of manufacturing sectors, for example, in the mattress, stainless steel, and beverage industries, where plants are spread out geographically. As an example, Sambasivan and Yahya (2005) study a real-world problem in a steel manufacturer.

Multi-plant lot sizing problems may be classified into one of two types. The *dependent* type are those whose plants need each other to produce items, i.e., the production environment has more than one stage and some items need other items from other plants to be produced (Bhatnagar et al., 1993; Wu and Golbasi, 2004; Kaminsky and

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Simchi-Levi, 2003). The *independent* type are those whose production centers are independent, i.e., the plants individually supply the items demanded (Sambasivan and Schmidt, 2002; Sambasivan and Yahya, 2005). In both cases, the transfer of lots within the plants are accounted for and the optimal solution to the problem involves production planning integrating the whole set of plants.

Few papers have previously addressed MPCLSP and few solution methods have been proposed. Sambasivan and Schmidt (2002) described a heuristic based on transfers of production lots. Although the authors described most of the parameters used in their computational tests, they are not clear in the definition of *loose* and *tight* capacities, which makes their experiments difficult to reproduce. Sambasivan and Yahya (2005) proposed a method based on Lagrangian relaxation. In computational tests, the authors observe that the mean gap of their solution with respect to the optimal is inversely proportional to the number of items. The configuration of instances in Sambasivan and Yahya (2005) was clear and we are able to reproduce their experiments on the same set of instances.

This paper proposes a greedy randomized adaptive search procedure (GRASP) heuristic embedded with a path-relinking strategy to find cost-effective solutions to the MPCLSP. The procedure for generating the initial solutions for the GRASP uses a greedy randomized version of the exact algorithm of Sung (1986) for the uncapacitated lot sizing problem with multiple machines. These initial solutions are usually infeasible, forcing us to apply transfer of lots between periods and plants to restore feasibility before applying the local search procedure. To analyze the performance of the heuristic, we designed three experiments. In the first experiment, we tested the heuristics using the instances proposed in Sambasivan and Yahya (2005). In the second experiment, our heuristics are tested using randomly generated instances according to Toledo and Armentano (2006). In both experiments, our results outperformed those of the literature. Finally, in the third experiment, we used the methodology proposed in Aiex et al. (2002, 2007) to assess experimentally the running time distributions of our randomized algorithms.

The paper is organized as follows. We begin by presenting the mathematical model in Section 2 and provide the algorithmic details in Section 3. Section 4 deals with the computational experiments. Finally, in Section 5 we conclude the paper.

2. Mathematical formulation

We review a mathematical model for the MPCLSP, based upon Sambasivan and Schmidt (2002). In this model, the terms $\forall i$, $\forall j$, and $\forall t$, indicate any element belonging to, respectively, sets *NI*, *MI*, and *TI* (which we describe below). This mixed-integer programming model is:

$$\min \sum_{i\in NI}\sum_{j\in MI}\sum_{t\in TI}\left(c_{ijt}x_{ijt}+s_{ijt}y_{ijt}+h_{ijt}I_{ijt}+\sum_{k\in MI,k\neq j} r_{jkt}w_{ijkt}\right)$$

subject to:

$$I_{ij,t-1}+x_{ijt}-\sum_{k\in MI,k\neq j} w_{ijkt}+\sum_{l\in MI,l\neq j} w_{iljt}-I_{ijt}=d_{ijt} \quad \forall i,\forall j,\forall t, \tag{1}$$

$$x_{ijt}\leqslant\left(\sum_{j\in MI}\sum_{l=t}^{T} d_{ijl}\right)y_{ijt} \quad \forall i,\forall j,\forall t, \tag{2}$$

$$\sum_{i\in NI}(b_{ijt}x_{ijt}+f_{ijt}y_{ijt})\leqslant C_{jt} \quad \forall j,\forall t, \tag{3}$$

$$I_{ij0}=0 \quad \forall i,\forall j, \tag{4}$$

$$x_{ijt},I_{ijt}\geqslant 0 \quad \forall i,\forall j,\forall t, \tag{5}$$

$$w_{ijkt}\geqslant 0 \quad \forall i,\forall j,\forall k,\forall t, \tag{6}$$

$$y_{ijt}\in\{0,1\} \quad \forall i,\forall j,\forall t, \tag{7}$$

where

T is the number of periods in the planning horizon;
N is the number of items in the planning horizon;
M is the number of plants in the planning horizon;
TI is the set composed by the elements $1,\ldots,T$;
NI is the set composed by the elements $1,\ldots,N$;
MI is the set composed by the elements $1,\ldots,M$;
d_{ijt} is the demand of item i at plant j in period t;
C_{jt} is the available capacity of production at plant j in period t;
b_{ijt} is the time to produce a unit of item i at plant j in period t;
f_{ijt} is the setup time to produce the item i at plant j in period t;
c_{ijt} is the unit production cost of item i at plant j in period t;
s_{ijt} is the setup cost of item i at plant j in period t;
h_{ijt} is the unit inventory cost of item i at plant j in period t;
r_{jkt} is the unity minimum transfer cost of an item from plant j to k in period t;
x_{ijt} is the quantity of item i produced at plant j in period t (variable);
I_{ijt} is the quantity of item i storage at plant j at the end of the period t (variable);
w_{ijkt} is the quantity of item i transferred from plant j to plant k during period t (variable);
y_{ijt} is a binary variable which assumes value 1 if the item i is produced at plant j in period t, and 0, otherwise (variable).

The minimum transfer cost r_{jkt} represents the minimum cost to transfer any item from plant j to plant k in period t and satisfies the triangle inequality $r_{ijt}+r_{lkt}\geqslant r_{jkt}$. The objective function encodes the goal of the optimization, which is the minimization of the total cost, i.e., production, setup, inventory, and transfer costs.

Constraints (1) refer to the inventory balance of the quantity of item i during period t at plant j. These constraints ensure that the demand of item i in period t at plant j is met by the production of this item in period t at plant j, added to the amount of the item stored in the previous period at the that plant and the quantity to be transferred from other plants to plant j, subtracted by the quantity of item i in period t that is transferred to the other plants and the quantity of item i that is stored in period t at plant j. Constraints (2) ensure that if item i is produced at plant j in period t, i.e., if $x_{ijt}>0$, then the binary variable $y_{ijt}=1$, which implies that the setup of the plant is to be considered. Constraints (3) ensure that the available capacity is not violated, while constraints (4) impose empty initial inventories. Finally, constraints Eqs. (5)–(7) impose the non-negativity of variables x, I, and w, and ensure that y variables are binaries.

The differences between the mathematical models of the MPCLSP and the CLSP with parallel machines are the non-existence of transfer costs in the objective function and the presence of a single warehouse in the CLSP model, which implies the unique inventory cost and production center.

3. Solution method

In order to find approximate solutions to the MPCLSP, we proposed two heuristics, a pure GRASP and a GRASP with path-relinking. GRASP was first proposed by Feo and Resende (1989, 1995). See also Resende and Ribeiro (2002) for a recent survey and Festa and Resende (2002) for a survey of a wide range of successful applications of GRASP.

Metaheuristics are high-level procedures specialized to solve combinatorial optimization problems. They guide other simpler heuristics to search for good-quality feasible solutions. GRASP is a metaheuristic based on a multi-start strategy, i.e., many initial solutions are generated through repeated applications of a