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Discrete Optimization

Balancing assembly lines with variable parallel workplaces: Problem definition and effective solution procedure

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ABSTRACT

Assembly line balancing problems (ALBP) arise whenever an assembly line is configured, redesigned or adjusted. An ALBP consists of distributing the total workload for manufacturing any unit of the products to be assembled among the work stations along the line subject to a strict or average cycle time. Traditionally, stations are considered to be manned by one operator, respectively, or duplicated in form of identical parallel stations, each also manned by a single operator. In practice, this assumption is usually too restrictive. This is particularly true for large products like cars, trucks, busses and machines, which can be handled by several operators performing different tasks at the same time. Only restricted research has been done on such parallel workplaces within the same station though they have significant relevance in real-world assembly line settings.

In this paper, we consider an extension of the basic ALBP to the case of flexible parallel workplaces (VWALBP) as they typically occur in the automobile and other industries assembling large products. The problem is defined and modelled as an integer linear program. As a solution approach a branch-and-bound procedure is proposed which also can be applied as a heuristic. Finally, computational experiments documenting the solution capabilities of the procedure are reported.

1. Introduction and literature review

Assembly lines are flow-oriented production systems which are typical in the industrial production of high quantity standardized commodities and even gain importance in low volume production of customized products. Among the decision problems which arise in managing such systems, assembly line balancing problems (ALBP) are important tasks in medium-term production planning (e.g. Baybars, 1986; Becker and Scholl, 2006; Boysen et al., 2007).

An assembly line consists of (*work*) *stations* $k = 1,\ldots,m$ arranged along a conveyor belt or a similar mechanical material handling equipment. The workpieces are consecutively launched down the line and are moved from station to station. At each station, certain operations are repeatedly performed regarding the *cycle time* (maximum or average time available for each workcycle). The decision problem of optimally partitioning (balancing) the assembly work among the stations with respect to some objective is known as the *assembly line balancing problem* (*ALBP*).

Manufacturing a product on an assembly line requires partitioning the total amount of work into a set $V = \{1,\ldots,n\}$ of *tasks* which constitute the nodes of a precedence graph. Performing a task i takes a *task time* (node weight) t_i. A *precedence relation* (i,j) means that task i must be finished before task j can be started and is contained in the arc set E of the precedence graph. Within the acyclical and topologically numbered graph redundant arcs are omitted. The precedence graph is described by the following sets with E^* denoting the transitive closure of E:

- $P_i = \{h \in V | (h,i) \in E\}$ set of direct predecessors of task $i \in V$
- $F_i = \{j \in V | (i,j) \in E\}$ set of direct successors (followers) of task $i \in V$
- $P_i^* = \{h \in V \mid (h,i) \in E^*\}$ set of all predecessors of task $i \in V$
- $F_i^* = \{j \in V \mid (i,j) \in E^*\}$ set of all successors of task $i \in V$

Any type of ALBP consists in finding a feasible *line balance*, i.e., an assignment of tasks to stations such that precedence constraints and possible further restrictions are fulfilled. The set S_k of tasks assigned to a station k $(=1,\ldots,m)$ constitutes its *station load*, the cumulated task time $t(S_k) = \sum_{j \in S_k} t_j$ is called station time. When a fixed common cycle time c is given, a line balance is feasible only if the *station time* of neither station exceeds c. In case of $t(S_k) < c$, the station k has an *idle time* of $c - t(S_k)$ time units in each cycle, i.e., it is repeatedly unproductive for this time span.

The most popular ALBP is called *Simple Assembly Line Balancing Problem* (*SALBP*). It has the following characteristics (cf. Baybars, 1986; Scholl and Becker, 2006; Boysen et al., 2007):

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Table 1
Versions of SALBP.

		Cycle time c	
		Given	Minimize
No. m of stations	Given	SALBP-F [$\|\|$]	SALBP-2 [$\|\|c$]
	Minimize	SALBP-1 [$\|\|m$]	SALBP-E [$\|\|E$]

(S-1) to (S-5): Mass-production of a homogeneous product on a serial line with fixed process observing a common fixed cycle time.
(S-6) Deterministic (and integral) task times t_j with $t_j \leqslant c$.
(S-7) No assignment restrictions of tasks besides precedence constraints.
(S-8) A task cannot be split among two or more stations.
(S-9) All stations are equally equipped with respect to machines and workers.

The general objective consists of maximizing the line efficiency Eff = $t_{\text{sum}}/(m \cdot c)$ with total task time $t_{\text{sum}} = \sum_{j=1}^{n} t_j$. Several problem versions arise from varying the objective as shown in Table 1. The tuple-notations specify the characterizations of the problem versions within the recent classification scheme of Boysen et al. (2007). All versions of SALBP and, thus, all generalizations like the new problem VWALBP, are NP-hard (cf. Wee and Magazine, 1982; Scholl, 1999, Chapter 2.2.1.5).

The basic SALBP has been extended and generalized in many research papers constituting a large number of generalized assembly line balancing problems (GALBP). For recent surveys and classifications of those problems see Becker and Scholl (2006) and Boysen et al. (2007).

Among these generalizations, the following are particularly relevant for the problem discussed here or helpful to explain what is different in the new problem. To focus on the problem itself, we only discuss assumptions and real-world backgrounds rather than algorithmic developments.

- *Parallel stations*: Two or more stations are arranged in parallel to allow for increased local cycle times (if p parallel stations are installed, the local cycle time amounts to $p \cdot c$). Then, it is possible to assign tasks with operation times greater than the cycle time. Because each copy of the station performs the *same set of tasks* in an alternating manner, it has *exclusive access* to the workpiece currently within its station area (cf. Buxey, 1974; Pinto et al., 1975, 1981; Sarker and Shanthikumar, 1983; Bard, 1989).
- *Assignment restrictions:* In practice, there are usually constraints which restrict the assignment of tasks to stations in addition to the cycle time constraint and the precedence relations (cf. Scholl, 1999, Chapter 1.3.4; Boysen et al., 2007; Scholl et al., 2008). Regularly, there are restrictions on assigning tasks to certain stations. Such *station restrictions* are particularly relevant when the line is already installed and should be rebalanced without rearranging all the equipment (cf. Tonge, 1960; Boguschewski et al., 1990), only certain station types are capable of performing some task (Johnson, 1983) or the workpiece is required in a certain position (Lapierre and Ruiz, 2004). Furthermore, there are often *incompatible tasks* that must not be assigned to the same station, e.g. due to the danger of soiling the seats of a car if the same worker has to handle the seats and to lubricate movable parts (cf. Agnetis et al., 1995; Johnson, 1983; Rachamadugu, 1991). Another reason for setting tasks incompatible is due to different *mounting positions* at a large workpiece (like cars, trucks, washing machines). In order to reduce walking distances of workers, certain combinations of mounting position and, thus, the corresponding tasks are defined to be incompatible (cf. Johnson, 1983).
- When large workpieces are assembled, it is possible that different operators work at the same product unit simultaneously (cf. Kilbridge and Wester, 1962; Akagi et al., 1983). That is, *several workplaces* are installed *at the same station*. Each worker (workplace) gets an own set of tasks at individual mounting positions. It has to be ensured that the workers do not interfere with each other by exclusively assigning each mounting position required to a single workplace. In case of a *two-sided line* (2ALBP), each station consists of (up to) two workplaces, one at the right and one at the left side of the line (Bartholdi, 1993; Kim et al., 2000; Lee et al., 2001; Lapierre and Ruiz, 2004; Lapierre et al., 2006). All tasks that have to be performed at the left (right) side of the workpiece must be assigned to a left (right) workplace. Remaining tasks can be assigned either to a left or a right workplace. Thus, the workpiece can be considered as being subdivided into two incompatible mounting positions (left and right), or alternatively, each pair of tasks which have to be performed on opposite sides is set incompatible. The two-sided line can be generalized to a *multi-sided line* (NALBP) by increasing the maximal number of workplaces per station (cf. Gehring and Boguschewski, 1990), e.g. a car can be subdivided into 4 exclusive mounting positions (front left, front right, back left, back right).

In Section 2, we generalize the concept of stations with multiple workplaces to consider real-world conditions as accurately as possible. The resulting problem is called VWALBP. Lower bounds and reduction rules are developed in Section 3. A branch-and-bound procedure for optimally solving this problem, which can also be applied as a heuristic, is presented in Sections 4 and 5. Computational experiments that evaluate the performance of the solution methods are reported in Section 6. The paper ends with conclusions and remarks on future research challenges in Section 7.

2. The assembly line balancing problem with variable workplaces (VWALBP)

As mentioned before, many products manufactured on assembly lines are large enough to be worked at by several workers simultaneously. As a major example, we focus on the final assembly of cars in the automobile industry where up to five workplaces are installed within a single station on a paced assembly line. Other examples of products assembled in such a manner are trucks, busses, large machines and even helicopters (Bartholdi, 1993; Lee et al., 2001).

2.1. Problem description

In the following, we define a new decision problem which is intended to model the situation of such car assembly lines (and related production systems as mentioned before) as realistically as possible. However, in order to define a basic problem, we do not include all conditions which might occur at a real line but focus on the most important ones giving structure to the problem. The new problem is called assembly line balancing problem with variable workplaces (VWALBP) and is an extension of SALBP characterized by the assumptions (S-1) to (S-5) and modifications of (S-6) to (S-9):

(V-6) *Task times*: Deterministic and (w.l.o.g.) integral task times t_j some of which might exceed the cycle time. Such *extra-long tasks* (collected in set EL) occur, e.g., when mounting large pieces like cockpits or examining the electrical devices already installed (see (V-8)).