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Optimal warranty policies for systems with imperfect repair

Wee Meng Yeo *, Xue-Ming Yuan

Singapore Institute of Manufacturing Technology, 71 Nanyang Drive, Singapore 638075, Singapore

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ABSTRACT

We investigate a system whose basic warranty coverage is minimal repair up to a specified warranty length. An additional service is offered whereby first failure is restored up to the consumers' chosen level of repair. The problem is studied under two system replacement strategies: periodic maintenance before and after warranty. It turns out that our model generalizes the model of Rinsaka and Sandoh [K. Rinsaka, H. Sandoh, A stochastic model with an additional warranty contract, Computers and Mathematics with Applications 51 (2006) 179–188] and the model of Yeh et al. [R.H. Yeh, M.Y. Chen, C.Y. Lin, Optimal periodic replacement policy for repairable products under free-repair warranty, European Journal of Operational Research 176 (2007) 1678–1686]. We derive the optimal maintenance period and optimal level of repair based on the structures of the cost function and failure rate function. We show that under certain assumptions, the optimal repair level for additional service is an increasing function of the replacement time. We provide numerical studies to verify some of our results.

1. Introduction

For any system that is sold under a free-repair warranty (FRW), the manufacturer will incur a cost during servicing. Under FRW, most of the cost modeling in the existing literature assumes that the system either undergoes perfect repair ("as good as new") or minimal repair ("as bad as old"). The concept of imperfect repair is a generalization of the two extreme cases mentioned. Throughout this paper, we denote $\delta \in [0,1]$ to be the level of repair chosen, where $\delta = 0$ and $\delta = 1$ correspond to minimal repair and perfect repair, respectively. We also assume that selecting $\delta = 0$ is the default warranty that is provided free to the consumers. Traditionally, imperfect repair is studied in the designing of optimal maintenance program for a system. Readers can refer to Pham and Wang [8] as a good source of references for imperfect maintenance. Some reasons for the interests in imperfect maintenance or repair are to reduce repairing of wrong parts and to avoid "over-repairing".

Warranty analysis under imperfect repair has recently received considerable attention among researchers. Chen and Popova [1] derive the optimal number of preventive maintenance (PM) that should be carried out to minimize the total expected cost during warranty for the manufacturer. Dagpunar and Jack [2] consider the imperfect maintenance in which the manufacturer proposes the level of recovery for the system during warranty period. The manufacturer will bear the cost for PM and minimal repairs. The cost is dependent on the age of the failed item and degree of repair while maintenance points may not be equal. The technique used is non-linear programming. Jain and Maheswari [4] propose a model for the optimal maintenance of the system under pro-rata warranty policy. If the system fails before warranty ω, it is replaced by the manufacturer and the warranty renews. This process continues until the system survives beyond ω, upon which the consumer is responsible for the maintenance. After ω, system undergoes periodic maintenance and is minimally repaired at each failure. The system is completely replaced after the Nth maintenance. The aim is to determine the optimal period and the number of PM points. Wang [11] discusses hardware warranty cost modeling from the manufacturer's point of view. He treats imperfect repair such that after repair the lifetime of a system will be reduced to a fraction $\alpha \in (0,1)$ of the one immediately preceding it, i.e, lifetime decreases as the number of repairs increases. Hence, the imperfectness of repair is inherent in the nature of the system rather than due to the consumer's choice. Yun et al. [16] introduce warranty servicing strategies using the level of imperfect repair as a decision variable. Many of the servicing strategies proposed earlier are based on minimal repair and replacement. Thus, their work by considering imperfect repair is an extension of the many previously studied servicing strategies. However, their model is again an optimization problem from the manufacturer's point of view.

* Corresponding author. Tel.: +65 67932177.
E-mail addresses: wmyeo@simtech.a-star.edu.sg (W.M. Yeo), xmyuan@SIMTech.a-star.edu.sg (X.-M. Yuan).

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Generally, there are relatively fewer literatures investigating consumer's optimal strategies given that extended warranty plans are offered. We will briefly mention works that are related to ours. Yeh et al. [13] analyzes the impact of renewing free-replacement warranty (RFRW) on the classical age-replacement problem. They derive the long-run average cost and determine the optimal replacement age under RFRW. Finally, they compare the result to the classical age-replacement problem without warranty. Chien and Chen [14], and Chien [15] extend the model of Yeh et al. [13], the analysis is carried out in a similar flavor. They assume that there is a probability of unplanned replacement and minimal repair when system fails before the preventive replacement periods. However, all these studies excludes the possibility of imperfect repairs.

Recently, Chien [10] extends the model of Yeh et al. [13] to include imperfect repair. He investigates the impact of an imperfect renewing free-replacement warranty on the age-replacement policy with an increasing failure rate. The product that fails during the warranty is replaced by another item whose reliability is less than the original. He derives the long-run average cost and determines the optimal age for preventive replacement periods.

This paper investigates a model which incorporates imperfect repair and extends the model of Yeh et al. [13] as well. However, one essential difference in our work compared to the work of Chien [10] is that imperfect repair is modeled as a choice for the consumer, according to a particular extended warranty program offered by the manufacturer. Our imperfect repair modeling is based on the class of ARA_1 (arithmetic reduction of age model with memory one, see Doyen and Gaudoin [12] for further details).

Our model also assumes the manufacturer offers the following enhanced warranty program that allows the consumer to select the level of repair, $\delta \in [0,1]$ under which,

- the manufacturer copes with the first failure according to δ chosen by the consumer,
- the manufacturer conducts minimal repairs at a downtime cost incurred by the consumer during the warranty period, and finally,
- the manufacturer performs minimal repair for each failure at a suitable fee after the warranty expires.

A similar model has been studied by Rinsaka and Sandoh [9]. However, they assume that the manufacturer always offers a full replacement whenever the consumer chooses to purchase the additional warranty. We allow the consumer to choose, thereby extending the model of Rinsaka and Sandoh [9]. In practice, we can think of a consumer who has a number of choices for the level of protection that he needs. The online purchasing of Dell computers has included the choices of additional warranty services which may present a dilemma to the consumers. The first objective is to derive sufficient conditions under which the optimal replacement time for the system exists. We also provide conditions helping the consumer to choose the optimal level of imperfect repair under the extended warranty program. Many previous works apply imperfect repair to maintenance, and the cost is usually borne by the manufacturer. As a result, this work leads to a variant of an extended warranty program often found in today's competitive market. The objective functions studied in this paper are the long-run average cost and the utility function, respectively. The outline of this paper is as follows. The model and notations are given in Section 2. Section 3 deals with the derivations of the long-run average cost model and utility function for preventive maintenance after the expiration of warranty. The consumer's optimal choice is also characterized via the property of the cost and failure rate function. Similar to Section 3, Section 4 derives the objective functions and other results for maintenance action before the expiration of warranty. In Section 5, we conduct some numerical studies to verify some of our results, including an experiment motivating the search for local joint minimum (τ^*,δ^*). We provide a concluding remark including some possible extensions to this work in Section 6. Finally, all derivations leading to the characterization of optimal repair choice in Section 3 can be found in Appendix.

2. Model

Let ω be the offered warranty length for the system. Let τ be the periodic replacement time of the system. When the system is under repair, during warranty period ω, C_d is the downtime cost borned by the consumer. Denote C_p to be the price of the system. After warranty ω, the consumer bears a unit cost of C_m for those minimal repair services. Random variable Y with c.d.f $F(y)$ and p.d.f $f(y)$ describes the lifetime of the system satisfying $F(0) = 0$, and $F(\infty) = 1$. Let $\overline{F}(x) = 1 - F(x)$ and $h(x)$ be the hazard function which describes the instantaneous failure rate at point $x \geqslant 0$, where $h(x) = \frac{f(x)}{1-F(x)}$. For the case of minimal repairs, the number of failures in $(0,u]$ is a non-homogeneous Poisson process (NHPP) with intensity function (see [5]) $H(u) = \int_0^u h(s)ds$. As the NHPP has stationary increments, the number of minimal repair in the interval $(a,b]$ is $H(b) - H(a)$. We assume that the failure rate is of increasing failure rate, i.e, $h(t)$ is increasing function for $t \in (0,\infty)$ and $H(t) = 0$ for $t \leqslant 0$. At an additional expense, the consumer is given a choice to "imperfectly" repair the failed item up to perfectly repairing the system. This is an additional service that provides an extra protection to the consumer. The decision is made at the beginning of each cycle ("cycle" defined to be the period between $[(n-1)\tau, n\tau]$ for all n). We shall assume that the strategy is the same for every period to simplicity. Let $\delta \in [0,1]$ be the chosen level of repair. Given that $\{Y = x\}$, we assume that the failure rate $h(x^+) = h((1-\delta)x^-)$. The resulting $h((1-\delta)x^-)$ is known as minimal wear intensity, i.e. a maximal lower bound for the failure intensity, for the ARA_1 model. Its existence is shown in Doyen and Gaudoin [12]. The cost of choosing this additional warranty service at service level δ is $c(\delta) + R_{\{\delta>0\}}$, where $R \geqslant 0$ is the fixed cost incurred by the consumer. We shall assume that $c(\delta)$ is continuous on $[0,1]$, in particular $\lim_{\delta\to 0^+} c(\delta) = 0$ for ease of analysis. In addition, we have factored in the fixed cost R when the consumer chooses imperfect repair. Yeh et al. [13] study a special case of our model when $\delta = 0$, the case when no additional warranty service is provided. When $\delta = 1$, $C_d = 0$, and $R = 0$, the additional warranty sought is the replacement (to be seen as unplanned) of a completely new system. Such a model has been considered in Rinsaka and Sandoh, see [9], where they study the optimal strategy of the consumer using the cost of the additional warranty and cost of minimally repairing the system after warranty. They have not factored in the downtime cost incurred during the minimal repair services. The objective function used in their paper is the utility function $U(x) = \frac{1}{\beta}(1 - e^{\beta x})$, $\beta > 0$ where x is the initial wealth. For the choice of this utility function, readers are advised to refer to Murthy and Asgharizadeh [6]. Hence, our model is seen as a natural extension to that of Yeh et al [13], as well as that of Rinsaka and Sandoh [9]. In the following sections, we attempt to provide cost modeling from the consumers' point of view. The strategies of the consumer include planned replacement before and after warranty while the objective functions are the long-run average cost and the utility function.