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Innovative Applications of O.R.

Multi-reservoir production optimization

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ABSTRACT

When a large oil or gas field is produced, several reservoirs often share the same processing facility. This facility is typically capable of processing only a limited amount of commodities per unit of time. In order to satisfy these processing limitations, the production needs to be *choked*, i.e., scaled down by a suitable *choke factor*. A *production strategy* is defined as a vector valued function defined for all points of time representing the choke factors applied to reservoirs at any given time. In the present paper we consider the problem of optimizing such production strategies with respect to various types of objective functions. A general framework for handling this problem is developed. A crucial assumption in our approach is that the *potential production rate* from a reservoir can be expressed as a function of the remaining recoverable volume. The solution to the optimization problem depends on certain key properties, e.g., convexity or concavity, of the objective function and of the potential production rate functions. Using these properties several important special cases can be solved. An *admissible production strategy* is a strategy where the total processing capacity is fully utilized throughout a plateau phase. This phase lasts until the total potential production rate falls below the processing capacity, and after this all the reservoirs are produced without any choking. Under mild restrictions on the objective function the performance of an admissible strategy is uniquely characterized by the state of the reservoirs at the end of the plateau phase. Thus, finding an optimal admissible production strategy, is essentially equivalent to finding the optimal state at the end of the plateau phase. Given the optimal state a backtracking algorithm can then used to derive an optimal production strategy. We will illustrate this on a specific example.

1. Introduction

Optimization is an important element in the management of large offshore exploration & production (E&P) assets, since many investment decisions are irreversible and finance is committed long-term. van den Heever et al. (2001) classify decisions made in reservoir management in two main categories, design decisions and operational decisions. Design decisions comprise selecting the type of platform, the staging of compression and assessing the number of wells to be drilled in a reservoir. These decisions are discrete in nature. In operational decisions production rates from individual reservoirs and wells are assessed. In contrast to design decisions, operational decisions are continuous in nature.

Neiro and Pinto (2004) propose a framework for modelling the entire petroleum supply chain. Ivyer and Grossmann (1998) present a multi-period mixed-integer linear programming formulation for the planning and scheduling of investment and operation in offshore oilfields. In other approaches a case and scenario analysis system is constructed for evaluating uncertainties in the E&P value chain, see Narayanan et al. (2003) for details. In Floris and Peersmann (2000), a decision scenario analysis framework is presented. Here, scenario and probabilistic analysis is combined with Monte Carlo simulation. Optimization can also be performed using a simulator, where real-time decisions are made subject to production constraints. Davidson and Beckner (2003) and Wang et al. (2002) use this technique. Their decision variables include binary on/off conditions and continuous variables. Uncertainty was not considered in these works. Optimization of oil and gas recovery is also a considerable research area, see Bittencourt and Horne (1997); Horne (2002) or Merabet and Bellah (2002).

Many of the contributions listed above focus on the problem of modelling the entire hydrocarbon value chain, where the purpose is to make models for scheduling and planning of hydrocarbon field infrastructures with complex objectives. Since this value chain is very complex, many aspects of it needs to be simplified to be able to construct such a comprehensive model.

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In the present paper we focus on the problem of optimizing production in an oil or gas field consisting of many reservoirs using the framework introduced in Haavardsson and Huseby (2007). This framework also includes uncertainty about key reservoir parameters. In this paper, however, the optimization problem will be analyzed deterministically. By focusing on only one specific part of the value chain, we are able to provide insight into how a large oil or gas field should be produced. Moreover, the methodology developed here can be used in a full scale value chain risk analysis.

As in Haavardsson and Huseby (2007), we assume that the field has been analyzed using state-of-the-art reservoir simulation methods. Based on the output from these simulations simplified *production profile models* for each of the reservoirs can be constructed and used as input to the optimization procedure. How to construct such profile models is described in Haavardsson and Huseby (2007). For a related approach see Li and Horne (2002).

We consider a situation where several reservoirs share the same processing facility. Oil, water and gas flow from each reservoir to this facility. The processing facility is only capable of handling limited amounts of the oil, gas and water per unit of time. In order to satisfy the resulting constraints, the production needs to be *choked*. That is, at any given point of time the production from each of the reservoirs are scaled down by suitable *choke factors* between zero and one, chosen so that the total production does not exceed the processing capacity. This situation is handled by introducing the concept of a *production strategy*. A production strategy is a vector valued function defined for all points of time $t \geqslant 0$ representing the choke factors applied to the reservoirs at time t. The problem is then reduced to finding a production strategy which is optimal with respect to a suitable objective function. In this setting, we focus on optimizing the oil production and leave the simultaneous analysis of oil, gas and water production for future work.

A fundamental model assumption is that the *potential production rate* from a reservoir, can be expressed as a function of the remaining recoverable volume. We refer to this function as the *potential production rate function* or *PPR-function* of the reservoir. It turns out that the PPR-functions play an important part in the analysis. Under some mild assumptions on the objective function it is proved that the performance of a production strategy is uniquely determined by the state of the reservoirs at the end of the plateau phase. Thus, an optimal production strategy can be found by first finding the optimal state of the reservoirs at the end of the plateau phase and then find a way to produce the reservoirs to reach this optimal state. It turns out that the optimal state of the reservoirs at the end of the plateau phase depends on the convexity of the PPR-functions and the quasi-convexity[1] of the objective function. Here two important situations are analyzed. To reach the optimal state of the reservoirs at the end of the plateau phase a backtracking algorithm is proposed.

2. Basic concepts and results

We consider the oil production from n reservoirs that share a processing facility with a constant process capacity $K > 0$, expressed in some suitable unit, e.g., kS m^3 per day. Let $\boldsymbol{Q}(t) = (Q_1(t), \ldots, Q_n(t))$ denote the vector of cumulative production functions for the n reservoirs, and let $\boldsymbol{f}(t) = (f_1(t), \ldots, f_n(t))$ be the corresponding vector of PPR functions. We assume that the PPR functions can be written as

$$f_i(t) = f_i(Q_i(t)), \quad t \geqslant 0, \; i = 1, \ldots n.$$

Note that this assumption implies that the potential production rate of one reservoir does not depend on the volumes produced from the other reservoirs. We will also assume for $i = 1, \ldots n$ that f_i is non-negative and non-increasing as a function of Q_i. These assumptions reflect the natural properties that the production rate cannot be negative and that the reservoir pressure typically decreases as more and more oil is produced. In order to ensure uniqueness of potential production profiles we also assume that f_i is Lipschitz continuous in Q_i.

In general the cumulative production functions $Q_1(t), \ldots, Q_n(t)$ depend on their respective PPR-functions as well as how the production from the field is run. Due to the limitation in the processing capacity, the total production rate from the field will typically be less than the sum of the potential rates for the reservoirs throughout the plateau phase of the field. The cumulative production functions are of course also limited by the recoverable volumes in the reservoirs. We denote these volumes by $V_1, \ldots, V_n$, respectively. As Q_i tends towards V_i, the potential production rate becomes smaller and smaller. In the limit we assume that $f_i(V_i) = 0, i = 1, \ldots, n$. In real life, however, the production is cut as soon as the total production rate becomes less than a certain cut-off rate typically determined by economic considerations.

A *production strategy* is defined as a vector valued function $\boldsymbol{b} = \boldsymbol{b}(t) = (b_1(t), \ldots, b_n(t))$, defined for all $t \geqslant 0$, where $b_i(t)$ represents the *choke factor* applied to the ith reservoir at time $t, i = 1, \ldots, n$. Notice that

$$0 \leqslant b_i(t) \leqslant 1, \quad i = 1, \ldots n, \; t \geqslant 0, \tag{2.1}$$

which means that the actual production rate cannot be increased beyond the potential production rate at any given point of time. We refer to the individual b_i-functions as the *choke factor functions* of the production strategy. The *actual production rates* from the reservoirs, after the production is choked is given by

$$\boldsymbol{q}(t) = (q_1(t), \ldots, q_n(t)),$$

where

$$q_i(t) = \frac{dQ(t)}{dt} = b_i(t) f_i(Q_i(t)), \quad i = 1, \ldots, n.$$

We also introduce the total production rate function $q(t) = \sum_{i=1}^{n} q_i(t)$ and the total cumulative production function $Q(t) = \sum_{i=1}^{n} Q_i(t)$. To reflect that $\boldsymbol{q}, q, \boldsymbol{Q}$, and Q depend on the chosen production strategy $\boldsymbol{b}$, we sometimes indicate this by writing $\boldsymbol{q}(t) = \boldsymbol{q}(t, \boldsymbol{b}), q(t) = q(t, \boldsymbol{b}), \boldsymbol{Q}(t) = \boldsymbol{Q}(t, \boldsymbol{b})$, and $Q(t) = Q(t, \boldsymbol{b})$. To satisfy the physical constraints of the reservoirs and the process facility, we require that the constraint (2.1) is satisfied and that

$$\sum_{i=1}^{n} b_i(t) f_i(Q_i(t), t) \leqslant K, \quad t \geqslant 0. \tag{2.2}$$

[1] For a definition of quasi-convex and quasi-concave functions see Appendix A.2.