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On the bias of Croston's forecasting method

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Abstract

Croston's forecasting method (CR) has been shown to be appropriate in dealing with intermittent demand items. The method, however, suffers from a positive bias as discussed by Syntetos and Boylan [Syntetos, A.A., Boylan, J.E., 2005a. The accuracy of intermittent demand estimates. International Journal of Forecasting 21, 303–314] who proposed a modification (SB). Unfortunately, the modification ignores the damping effect on the bias of the probability that a demand occurs. This leads to overcompensation and a negative bias, which can in fact be larger than the positive bias of the original method. Syntetos [Syntetos, A.A., 2001. Forecasting for Intermittent Demand, Unpublished Ph.D thesis, Buckinghamshire Chilterns University College, Brunel University] proposed another modification (SY) that takes the damping effect into account, thereby reducing the bias. However, he eventually disregarded it from the empirical analysis, because of the analytical results that SY never dominates SB as well as CR when both bias and variance are considered. Levén and Segerstedt [Levén, E., Segerstedt, A., 2004. Inventory control with a modified Croston procedure and Erlang distribution. International Journal of Production Economics 90, 361–367] also proposed a modified Croston method (LS) and claimed it to be unbiased. We compare all four methods in a numerical study. Our results strengthen the finding from Boylan and Syntetos [Boylan, J.E., Syntetos A.A., 2007. The accuracy of a modified Croston procedure. International Journal of Production Economics 107, 511–517] that LS suffers from a much more severe bias that the other methods. They also confirm SB as the best method when the Mean Square Error is considered. However, SY has a much smaller average absolute bias of 1% compared to 5% for the SB method. From an inventory control point of view, this is an important advantage of the SY method, since biases distort calculations of the expected lead time demand as well as safety stock calculations. An additional advantage of the SY method is its robust performance over the range of parameter values that we considered. Based on these results, we suggest that the SY method should receive more consideration as an alternative to CR and SB.

Keywords: Forecasting; Intermittent demand; Croston's method

1. Introduction

It is not easy to forecast intermittent demand due to its erratic and sometimes lumpy nature. Many organisations in the manufacturing and especially service industries simply use single exponential smoothing. However, as was first shown by Croston (1972), this generally leads to inappropriate stock levels. Croston proposed an alternative method (abbreviated as CR) that takes account of both demand size and inter-arrival time between demands. The method is now widely used in industry and it is incorporated in various best selling forecasting software packages (see Syntetos and Boylan, 2005a).

The CR method has been assessed by several authors since 1972. The literature is reviewed in detail in Section 2. Most authors come to the conclusion that the CR method is more suitable for intermittent demand than traditional methods such as moving average and single exponential smoothing. In fact, as shown by Teunter and Duncan (2008), contradictory results can be explained by the use of inappropriate performance measures.

A disadvantage of the CR method is that it is positively biased. Syntetos and Boylan (2001) noted this and proposed a modification (abbreviated as SB). However, as we will show in Section 3, that modification over-compen-

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sates, leading to a negative bias instead. Syntetos (2001) also noted this and propose a 'corrected' approximately unbiased method (abbreviated as SY). Levén and Segerstedt (2004) further proposed a method that they claimed to be unbiased (abbreviated as LS), but their method is even more biased as we also show in Section 3.

In this study, we compare CR, SB, SY and LS in a numerical study. The remainder of the manuscript is organized as follows. A detailed account of the literature is given in Section 2 and the details of all methods are presented in Section 3. Results of the numerical study are discussed in Section 4. The paper ends with conclusions in Section 5.

2. Literature on Croston's method

We will review the contributions chronologically. Rao (1973) made corrections to several expressions in Croston's paper without affecting the final conclusions or the forecasting procedure. Schultz (1987) presented a forecasting procedure, which is basically the CR method and suggested a base-stock inventory policy with replenishment delays. We note that Schultz (1987) proposed the use of two smoothing parameters (one for demand size, the other for demand intervals), whereas in the original paper by Croston (1972) a common smoothing parameter was assumed. Willemain et al. (1994) compared the CR method with exponential smoothing and concluded that the CR method is robustly superior to exponential smoothing, although results with real data in some cases show a more modest benefit. Johnston and Boylan (1996) obtained similar results, but further showed that the CR method is always better than exponential smoothing when the average inter-arrival time between demands is greater than 1.25 review intervals. Sani and Kingsman (1997) compared various forecasting and inventory control methods on some long series of low demand real data from a typical spare parts depot in the UK. They concluded based on cost and service level, that the best forecasting method is moving average followed by the CR method.

An important contribution is that by Syntetos and Boylan (2001). They showed that the CR method leads to a biased estimate of demand per unit time. They also propose a modified method (SB) and demonstrate the improvement in a simulation experiment.

Syntetos (2001) noted that the SB method is still biased and proposed another modification (the SY method). However, he eventually disregarded it from his empirical analysis, because of the analytical results that the SY method never dominates the SB method as well as the CR method when both bias and variance are considered.

Snyder (2002) critically assessed the CR method with a view to overcome certain implementation difficulties on the data sets used. Snyder made corrections to the underlying theory and proposed modifications. Ghobbar and Friend (2003) compared various forecasting methods using real data of aircraft maintenance repair parts from an airlines operator. The data is sporadic in nature and they showed that moving average, Holt's method and the CR method are superior to other methods such as exponential smoothing. Willemain et al. (2004) compared various forecasting methods using large industrial data sets. They showed that the bootstrapping method produces more accurate forecasts than both exponential smoothing and the CR method.

In an attempt to develop a forecasting procedure that can handle both fast moving and slow moving items, Levén and Segerstedt (2004) proposed a modification of the CR method which was thought to avoid the bias indicated by Syntetos and Boylan, 2001. The modification was shown to outperform exponential smoothing based on a simulation experiment. Eaves and Kingsman (2004) compared various forecasting methods using real data from the UK's Royal Air Force. They showed that the SB method is the best forecasting method for spare parts inventory control.

In an attempt to further confirm the good performance of their SB method, Syntetos and Boylan (2005a) carried out a comparison of forecasting methods including theirs and the original CR method. A simulation exercise was carried out on 3000 products from the automotive industry with "fast intermittent" demand. It was shown that the modification is the most accurate estimator. In another study, Syntetos et al. (2005) analyzed a wider range of intermittent demand patterns and made a categorisation to guide the selection of forecasting methods. They indicated that there are demand categories that are better used with the CR method and there are others that go well with the SB method.

A recent comparison by Syntetos and Boylan (2006) shows overall superior performance of the SB method, followed by moving average and the CR method. Another comparative study was conducted by Teunter and Duncan (2008), using a large data set from the UK's Royal Air Force. Using a new performance measure that compares target to achieved service level, they showed that CR, SB and LS all outperform moving average and exponential smoothing.

3. Theoretical background

Croston's original method (CR) forecasts separately the time between consecutive transactions p_t and the magnitude of the individual transactions z_t. At the review period t, if no demand occurs in a review period then the estimates of the demand size and inter-arrival time at the end of time t, $\hat{z}_t$ and $\hat{p}_t$, respectively, remain unchanged. If a demand occurs so that $z_t > 0$, then the estimates are updated by

$$\hat{z}_t = \alpha z_t + (1-\alpha)\hat{z}_{t-1},$$
$$\hat{p}_t = \alpha p_t + (1-\alpha)\hat{p}_{t-1},$$

where α is a smoothing constant between zero and one. Hence, the forecast of demand per period at time t is given as