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Importance sampling for integrated market and credit portfolio models

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Abstract

A sophisticated approach for computing the total economic capital needed for various stochastically dependent risk types is the bottom-up approach. In this approach, usually, market and credit risks of financial instruments are modeled simultaneously. As integrating market risk factors into standard credit portfolio models increases the computational burden of calculating risk measures, it is analyzed to which extent importance sampling techniques previously developed either for pure market portfolio models or for pure credit portfolio models can be successfully applied to integrated market and credit portfolio models. Specific problems which arise in this context are discussed. The effectiveness of these techniques is tested by numerical experiments for linear and non-linear portfolios.

Keywords: Bottom-up approach; Credit risk; Importance sampling; Interest rate risk; Risk management; Value-at-risk

1. Introduction

Due to their business activities, banks are exposed to many different risk types. Among these risk types are credit risk, market risk, operational risk, and business risk. The task of the risk management division is to measure all these risks and to determine the necessary amount of economic capital which is needed as a buffer to absorb unexpected losses associated with each of these risks. Most frequently, economic capital is understood as a Value-at-Risk (VaR) number. Thus, it is the amount of capital needed to absorb unexpected losses within a given time period up to a specified probability.

Predominantly, the necessary amount of economic capital is determined for each risk type separately. That is why the problem arises how to combine these various amounts of capital to a single number. Within the so-called building–block approach stipulated by the regulatory authorities, the amount of regulatory capital, which the banks have to hold for the different risk types, are just added. This is a quite conservative approach because it ignores diversification effects between the risk types. As a consequence, in general, the true amount of economic or regulatory capital that is needed is overestimated.

However, the alternative, namely to consider diversification effects to some extent, requires to model the stochastic dependence between the various risk types. In practice, some kind of heuristics, based on strong assumptions, are often used to merge the economic capital figures for the various risk types into one overall economic capital figure.[1] A theoretical more sound approach is to link the separately determined marginal distributions of losses resulting from different risk types by Copula functions (see, e.g. Ward and Lee (2002), Dimakos and Aas (2004), Rosenberg and Schuermann (2006)). However, the difficulty is to choose the correct Copula function, especially given the limited access to adequate data.

Another more sophisticated approach is to build up models for various risk types by integrating a specific risk type into existing models for the measurement of another risk type. This approach is pursued in this paper, which deals, more

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[1] For an overview on risk aggregation methods used in practice, see Joint Forum (2003), Bank of Japan (2005), and Rosenberg and Schuermann (2006).

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specifically, with the integration of market risk into credit portfolio models. Integrated market and credit portfolio models allow to determine simultaneously, in one common framework, the necessary amount of economic capital needed for the market risk and for the credit risk of banking book instruments, whereby possible stochastic dependencies between these two risk components can be taken into account directly. This latter approach is called a bottom-up approach, whereas the Copula-based approach represents a top-down technique.

For measuring the credit risk inherent in a banking book, a range of models has been developed. Well-known examples are CreditMetrics by J.P. Morgan Chase, CreditPortfolioView by McKinsey, Portfolio Manager by KMV, or CreditRisk$^+$ by Credit Suisse First Boston. A typical shortcoming of most credit portfolio models is that relevant market risk factors, such as risk-free interest rates or credit spreads, are not modeled as stochastic variables and hence are ignored during the revaluation of the credit sensitive instruments at the risk horizon. An exception is the approach Algo Credit developed by the risk management firm Algorithmics (see Iscoe et al. (1999)). Even the Basel II proposals do not regulate the interest rate risk of the banking book in a quantitative way, but only qualitatively under pillar II (see Basel Committee on Banking Supervision (2005)). In a typical credit portfolio model, fixed-income instruments, such as bonds or loans, are revalued at the risk horizon using the current forward rates and (rating-specific) forward credit spreads for discounting future cash flows. Even for derivatives with counterparty risk, only single values, so-called loan equivalents, are employed per possible rating grade of the counterparty at the risk horizon. Thus, the stochastic nature of the instrument's value in the future that results from changes in factors other than credit quality is ignored. This may cause an underestimation of the riskiness of the credit portfolio (see, e.g. Barnhill and Maxwell (2002), Kiesel et al. (2003), Grundke (2005)). An additional consequence is that correlations between changes of the debtors' credit quality and changes of market risk factors and hence the exposure at default cannot be integrated into the credit portfolio model. This is especially a problem for market-driven instruments, such as interest rate derivatives, because the exposure at default mainly depends on the stochastic evolution of the underlying market risk factors. Finally, ignoring relevant market risk factors in credit portfolio models, correlations between the exposures at default of different instruments, which depend on the same or correlated market risk factors, cannot be modeled, either.

However, adding market risk factors as additional ingredients of a credit portfolio model, the computational burden of calculating risk measures increases because the revaluation of the instruments at the risk horizon becomes more complex. Most standard credit portfolio models rely on Monte Carlo simulations for calculating the probability distribution of the future credit portfolio value.[2] This is already computer-time-consuming for standard credit portfolio models, especially for inhomogeneous portfolios with many obligors and when percentiles corresponding to high confidence levels have to be estimated. Thus, the need for efficient methods for calculating credit risk measures becomes even more pressing for integrated market and credit portfolio models.

For standard credit portfolio models, various efficiency enhancing computational approaches have been developed meanwhile. Among these are, for example, approaches based on Monte Carlo simulations combined with variance reduction techniques, mainly importance sampling (IS) (for a literature review see the next Section 2), Fourier-based approaches (see Duffie and Pan (2001), Merino and Nyfeler (2002), Reiß (2003)), computational approaches based on saddlepoint approximations (see, e.g. Arvanitis et al. (1998), Martin et al. (2001a,b), Gordy (2002), Barco (2004)), or methods which rely on the assumption that the portfolio is sufficiently large or sufficiently granular so that by the virtue of the (strong) Law of Large Numbers (or the Central Limit Theorem) approximations of the credit portfolio loss variable are possible (see, e.g. Finger (1999), Vasicek (1991, 2002), Gordy (2003)).

For integrated market and credit portfolio models, it might suggest itself to simply adjust and to apply these techniques also to this extended class of portfolio models. However, it has already been reported in the literature that this simple strategy does not always work. For example, Grundke (2007) finds that the Fourier-based approach when applied to an integrated market and credit portfolio model for estimating risk measures does not perform better than a crude Monte Carlo simulation. In this paper, we analyze the performance of IS as a special variance reduction technique. Monte Carlo simulations combined with IS are reported to be very flexible in the computation of overall risk measures as well as individual risk contributions. We transfer an IS technique previously developed for pure market portfolio models as well as an IS technique previously developed for pure credit portfolio models to the case of integrated market and credit portfolio models. We find that, when applying these techniques to the extended class of integrated market and credit portfolio models, specific problems arise which reduce their benefit. These problems are discussed and the effectiveness of these techniques is tested by numerical experiments for linear and non-linear portfolios.

The paper is structured as follows: in Section 2, an overview on related literature is given. In Section 3, a general framework for an integrated market and credit portfolio model is presented. Besides, a concrete specification of this general model is described, which afterwards is used for the numerical experiments. In Section 4, two IS techniques are applied to the general integrated market and credit portfolio model. The first one is a two-step-technique originally developed

[2] A prominent exception is the model CreditRisk$^+$ where due to specific assumptions the loss distribution can be computed by recursion.