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Decision Support

Performance evaluation: An integrated method using data envelopment analysis and fuzzy preference relations

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Abstract

In a multi-attribute decision-making (MADM) context, the decision maker needs to provide his preferences over a set of decision alternatives and constructs a preference relation and then use the derived priority vector of the preference to rank various alternatives. This paper proposes an integrated approach to rate decision alternatives using data envelopment analysis and preference relations. This proposed approach includes three stages. First, pairwise efficiency scores are computed using two DEA models: the CCR model and the proposed cross-evaluation DEA model. Second, the pairwise efficiency scores are then utilized to construct the fuzzy preference relation and the consistent fuzzy preference relation. Third, by use of the row wise summation technique, we yield a priority vector, which is used for ranking decision-making units (DMUs). For the case of a single output and a single input, the preference relation can be directly obtained from the original sample data. The proposed approach is validated by two numerical examples.

Keywords: Performance evaluation; Data envelopment analysis (DEA); Preference relations; Cross evaluation

1. Introduction

In a multi-attribute decision-making (MADM) situation the decision maker (DM) is faced with the question of which decision making unit (DMU) to adopt from among a set of alternative DMUs that are available to him. To model this problem, one typical method is to ask the DM to provide his preferences over a set of evaluated decision alternatives and construct preference relations using his expressed pairwise comparison information. The DM then solve this MADM problem by ranking all the evaluated DMUs based on the priority vector derived from a consistency judge matrix. There are usually two widely used preference relations: multiplicative preference relation (Saaty, 1980; Herrera et al., 2001; Chiclana et al., 2001) and fuzzy preference relation (FPR) (Orlovsky, 1978; Nurmi, 1981; Kacprzyk, 1986; Tanino, 1990). We present the definitions of two preference relations in Appendix II. FPR is preferred to multiplicative preference relation when MADM with incomplete information is presented to the DM (Chiclana et al., 2007). Both preference relations are based on pairwise comparison and thus incur some common research issues, e.g., the construction of preference relations (Vargas, 1990; Gheorghe et al., 2005) and the consistency problem of preference relations (Herrera-Viedma et al., 2004).

Either the multiplicative or the fuzzy preference relation is actually constructed based on a self-rated scheme (indicated by the diagonal elements in the preference matrix) and a cross-rated scheme (indicated by the non-diagonal elements). Classical techniques used to construct a preference relation are based on subjective evaluation, requiring much involvement of expert knowledge and time. An objective technique can greatly reduce the cost incurred

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by the involvement of expert knowledge and time in the evaluation process. DEA provides a tool for objective evaluation (Charnes et al., 1978). DEA is a nonparametric programming technique used to treat problems of multiple inputs and outputs associated with multiple DMUs. DEA is used to establish a best practice group from among a set of observed units and to identify the units that are inefficient when compared to the best practice group. Researchers have discussed the similarity of performance measurement and decision making problems using DEA (Doyle and Green, 1994; Parkan, 2006).

The purpose of the present paper is two-fold: (a) to introduce an integrated DEA/FPR method for ranking the DMUs in a decision making situation, and (b) to develop an alternative tool for construction of relation preference and extend the traditional DEA models by proposing a cross-evaluation method for use in performance evaluation.

The proposed approach includes three stages. The first stage is to run the CCR model and the proposed cross-evaluation DEA model to yield pairwise efficiency scores. The pairwise efficiency scores are then used to construct the fuzzy preference relation at the second stage. At the last step, we use the row wise summation technique to yield the priority vector for ranking DMUs. Similar technique was used in Sinuany-Stern et al. (2000) to address the relationship between DEA and AHP. Sinuany-Stern et al.'s model may not be practical due to two drawbacks. First, as indicated in their own work, "...we receive many efficient values, especially as the number of inputs and outputs increases..." (Sinuany-Stern et al., 2000; p. 115), which reflects a weak diagnostic power of their model. This is always a hurdle for their model since their model violates the DEA rule of thumb by allowing only two DMUs to be involved each time they run DEA. The rule of thumb requires in data the number of DMUs is no less than three times of the total number of input and output variables (Cooper et al., 2000). Second, by selecting only two DMUs in each DEA run, a total number of $2n(n-1)$ linear programs have to be solved, involving $n(n-1)$ programs for the CCR version (self-rated problem) and another $n(n-1)$ programs for the cross-rated problem, which obviously result in a heavy computation task. Our proposed method directly applies n DMUs of interests to CCR DEA and the revised benevolent DEA. Computation complexity is greatly reduced and diagnostic power is improved comparing to Sinuany-Stern et al.'s model. These advantages are to be discussed in both Sections 2 and 5.

The rest of this paper unfolds as follows. Section 2 presents the methodology to construct preference relations. An algorithm for alternative evaluation by use of the constructed preference relation is designed in Section 3. Section 4 shows how to construct preference relations directly from the original sample data in the case of single output and single input. Section 5 gives two illustrative examples, and finally, concluding remarks and further consideration are presented in Section 6.

2. Methodology

In this section, the preference relation is constructed by implementing a three-stage methodology. Note that we argue in the introduction that a preference relation is actually constructed based on a self-rated scheme and a cross-rated scheme, thus we need to establish self-rated and cross-rated problem by use of DEA at first. Hence, of the three stages, first we yield pairwise efficiency scores using two DEA models: the CCR model and the proposed cross-evaluation DEA model. The resulting pairwise efficiency scores are then utilized to construct the fuzzy preference relations at the second stage. At the last stage, by use of the row wise summation technique, the priority vector for ranking DMUs is obtained.

2.1. Paired DEA: CCR and cross-evaluation DEA

Suppose there are n DMUs, denoted as DMU_l $(l=1,2,\ldots,n)$ to be evaluated. Each DMU_l has m different inputs x_{il} $(i=1,2,\ldots,m)$ and s different outputs y_{rl} $(r=1,2,\ldots,s)$. Let the observed input and output vectors of DMU_l be $X_l=(x_{1l},x_{2l},\ldots,x_{ml})^{\mathrm{T}}>0$, $l=1,2,\ldots,n$, and $Y_l=(y_{1l},y_{2l},\ldots,y_{sl})^{\mathrm{T}}>0$, respectively, where "T" denotes the transpose. DEA determines for each alternative its efficiency value as the maximum of the ratio of its weighted scores for output criteria to weighted scores for input criteria under the constraint that this efficiency is bound from above by unity for all the alternatives of interest. This is known as the CCR DEA as a fractional programming problem (Charnes et al., 1978), which is then transformed into the following linear programming model (1) by use of the Charnes–Cooper transformation (Charnes and Cooper, 1962).

CCR DEA (self-rated problem)

$$\begin{aligned} E_{dd}=(\theta_d^{\mathrm{CCR}}) = \mathrm{Max} \quad & \mu_d^{\mathrm{T}} Y_d \\ \text{s.t.} \quad & \omega_d^{\mathrm{T}} X_l - \mu_d^{\mathrm{T}} Y_l \geqslant 0, \quad l=1,2,\ldots,n, \\ & \omega_d^{\mathrm{T}} X_d = 1, \\ & \omega_d^{\mathrm{T}} \geqslant 0, \quad \mu_d^{\mathrm{T}} \geqslant 0, \end{aligned} \tag{1}$$

where DMU_d is under evaluation and ω_d, μ_d are the associated input and output weight vectors. While DMU_d $(d=1,2,\ldots,n)$ is changed in the above CCR model n times, each for one DMU, the optimal weights $(\omega_d^{*\mathrm{T}},\mu_d^{*\mathrm{T}})$ and optimal efficiency E_{dd} given to DMU_d $(d=1,2,\ldots,n)$ are obtained. In the CCR model, each DMU optimizes the most favorable weights and receives its most favorable evaluation relative to any other unit. In other words, each DMU is self-evaluated. DMU_d is termed weakly efficient if and only if the optimal objective is equal to 1, i.e., $E_{dd}=1$. The cross-efficiency of DMU_j using the optimal weight of DMU_d is calculated as

$$\frac{\mu_d^{*\mathrm{T}} Y_j}{\omega_d^{*\mathrm{T}} X_j} \quad (d,j=1,2,\ldots,n).$$