Available online at www.sciencedirect.com
ScienceDirect
European Journal of Operational Research 192 (2009) 808–823
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Optimal policy for an inventory system with backlogging and all-units discounts: Application to the composite lot size model

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Received 11 September 2006; accepted 5 October 2007
Available online 23 October 2007

Abstract

This paper examines an inventory model with full backlogging and all-units quantity discounts. The practical scenario of a salesperson offering compensation to a client so as not to lose the sale is considered. The cost of a backorder thus includes both a fixed cost and a further cost which is proportional to the length of time the said backorder exists. A first algorithm is developed to determine the optimal policy while some extensions to this algorithm are obtained that include additional conditions on the model. In particular, the well known composite lot size model, developed by Tersine, is solved, incorporating a new stockout cost and a new all-units discount. Numerical examples are provided to illustrate the application of the algorithms.

Keywords: Logistics; Inventory model; Backlogging; Price discounts; Freight rates; Inspection discounts

1. Introduction

Inventory control of goods or products is a very important part of logistic systems, common to all economic sectors such as agriculture, industry, trade and business. Very large costs are incurred as a result of replenishment actions, shortages and the use of managerial and clerical time in making and routinely implementing inventory management decisions. Thus, properly designed decision rules, based on mathematical modeling, can lead to substantial benefits. The major problem in inventory control can be summed up by the two fundamental questions (i) when should a replenishment order be placed? and (ii) how much should be ordered?

The economic order quantity (EOQ) model proposed by Harris (1913) and subsequently by Wilson (1934) was the initial lot size model based on cost minimization (see, e.g. Lee and Nahmias, 1993, p. 4). Many extensions of Harris' EOQ model have been constructed and solved through the formulation of different assumptions. Among others are the cases where shortage is permitted. One group of these models assumes that, in the case of a shortage, the customer waits for the delivery of the next order for his demand to be fulfilled (the so-called *backorders* case). In the other case, the customer is not prepared to wait if there is a shortage (the so-called *lost sales* case). In the backorders case, the shortage cost may depend on the quantity of time, or shortage, or both, or it may be a fixed cost (Chikán, 1990, p. 27).

In Harris' EOQ model the unit purchasing cost of an item is assumed to be independent of the order size, and hence is excluded from consideration in the cost function. When the unit purchasing cost of an inventory system depends on the quantity of an order, it is referred to as a system with *quantity discounts.* Two types of quantity discounts are commonly

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0377-2217/$ - see front matter
doi:10.1016/j.ejor.2007.10.026

used: the all-units quantity discounts, in which the price of *all* units is discounted when the order size exceeds a critical level, and the *incremental* discounts, in which the discount price is only applied to the units in excess of the critical level. Many authors (Hadley and Whitin, 1963; Naddor, 1966; Love, 1979; Silver et al., 1998; Axsäter, 2000; Zipkin, 2000; Ghiani et al., 2004 and others) have discussed these problems in the context of Harris' EOQ model. The suggested solutions procedures usually involve some cost function evaluation as part of an algorithm.

New extensions have recently been carried out in different lines of research. When shortages are not allowed (Tersine and Toelle, 1985; Tersine et al., 1989; Hwang et al., 1990; Tersine and Barman, 1991, 1994 and many others), the unit discount from suppliers and/or freight discounts from shippers have been studied. In Aucamp (1982), an EOQ model is considered where the total freight cost depends on an integer number of carloads required to fill an order. In Jucker and Rosenblatt (1985) and Knowles (1988), quantity discounts are considered in the context of the newsboy problem. The dynamic lot size model with quantity discount in purchase price is dealt with in Federguen and Lee (1990). Benton (1991) considers quantity discount procedures under the conditions of multiple items, resource limitations and multiple suppliers. Burwell et al. (1997) incorporate quantity and freight discounts when price determines demand, but shortage is not allowed. Lewis (1998) establishes a practical range of price discounts for when purchasing orders larger than the economic order quantity (EOQ) are considered and shortage is not allowed. Ertogral et al. (2007) develop two models that integrate the transportation cost explicitly in the single-vendor single-buyer problem. They assume all-unit-discount transportation cost in the first model and the option of over declaration a shipment in the second model.

In Fazel et al. (1998), the inventory cost of purchasing under the economic order quantity model with quantity discount is determined and compared to the costs under the just-in-time model. The problem of stocking inventories of multiple items that share a common resource, such as warehouse space, where the vendor of the items offers quantity discounts, is considered in Güder and Zydiak (2000). An EOQ model in which the purchase price is described as a decreasing function of ordering quantity is studied in Matsuyama (2001). Swenseth and Godfrey (2002) present a model with two freight rate functions into the total annual logistics cost function to determine their impact on purchasing decisions. These authors do not consider the possibility of shortage. Rubin and Benton (2003) study the purchasing decisions facing a buying firm which receives incrementally discounted price schemes for a group of items in the presence of constraints such as budgets and space limitations.

The case of the EOQ model with full backlogging and discounts is considered in Hadley and Whitin (1963), Tersine (1994) and Tersine et al. (1995). These authors suppose that the unit shortage cost depends on the shortage time. In the first, a study of the all-units quantity discounts case is sketched and, in the incremental-discounts case, only a few indications are given. In Tersine (1994) and Tersine et al. (1995), the two types of discounts are studied. For the all-units discounts case a similar algorithm to the EOQ model without shortage is obtained. For the incremental-discounts case it is necessary to solve a quadratic equation at each step of the algorithm. Wee (1999) develops a deterministic inventory model with quantity discount, pricing and partial backordering when the product in stock deteriorates with time. We refer the reader to Benton and Park (1996) for a more detailed review of the related literature.

In this paper we study an EOQ model with all-units quantity discounts, where shortage is allowed and fully backlogged. The total cost function is developed using four general costs: order cost, purchasing cost, holding cost and shortage cost. Order cost is fixed per replenishment and holding cost is based on average stock. In shortage cost we include two significant costs: a fixed cost and a cost depending on the shortage time, that is, we consider the practical situation when a salesperson offers compensation to a client so as not to lose the sale, including a fixed cost and a cost which is proportional to the length of time the backorder exists. Moreover, that the purchase unit price depends on the quantity ordered is admitted. The optimal policy is obtained through a sequential optimization procedure in two stages that relies on a quadratic function (first stage) and a two piece function which includes the objective function of Harris' EOQ model and another more complex function (second stage). Firstly, a general procedure is developed and, relying on this procedure, an algorithm is obtained. Extensions of this algorithm are shown in the following cases: (i) when the minimum quantity to be purchased is positive and/or the maximum quantity is finite; (ii) when the lot size quantity must be an integer number and (iii) when the two previous conditions are jointly considered. Moreover, as a particular case, we solve the composite lot size model now including a fixed cost per unit backordered and three types of discount: quantity, freight and inspection. The inclusion of discounts for inspections is suggested by Tersine et al. (1992). Numerical examples are provided to illustrate the theory and the application of the algorithms. A different model, similar to the one in this article, but for the case where the backlogging cost only includes a fixed cost, is studied in San-José and García-Laguna (2003).

The paper is structured as follows. Section 2 introduces the assumptions and the notation. Section 3 describes the model and gives a general perspective for its resolution. Section 4 presents some properties to characterize the optimal policy and expose a general procedure to determine this policy. Section 5 presents an algorithm to determine the optimal policy, some extensions of which are shown in Section 6. A new approach to studying the composite lot size model is presented in Section 7. Several numerical examples to illustrate the application of the algorithms are provided in Section 8. Finally, the conclusions are given in Section 9.